

## Relaxation of a spring with an attached granular damper

Jonathan E Kollmer<sup>1</sup>, Achim Sack, Michael Heckel  
and Thorsten Pöschel

Institute for Multiscale Simulation, Friedrich-Alexander-Universität  
Erlangen-Nürnberg, Germany  
E-mail: [jonathan.kollmer@cbi.uni-erlangen.de](mailto:jonathan.kollmer@cbi.uni-erlangen.de)

*New Journal of Physics* **15** (2013) 093023 (9pp)

Received 8 May 2013

Published 13 September 2013

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/15/9/093023

**Abstract.** The oscillation of a spring may be attenuated by means of a granular damper. In difference to viscous dampers, the amplitude decays nearly *linearly* in time up to a finite value, from there on it decays much slower. We quantitatively explain the linear decay, which was a long-standing question.

### Contents

<b>1. Introduction</b>	<b>2</b>
<b>2. Linear decay of the amplitude</b>	<b>2</b>
<b>3. Modes of excitation</b>	<b>2</b>
<b>4. Linear relaxation of a damped spring</b>	<b>4</b>
<b>5. Conclusion</b>	<b>7</b>
<b>Acknowledgments</b>	<b>8</b>
<b>References</b>	<b>8</b>

<sup>1</sup> Author to whom any correspondence should be addressed.



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](http://creativecommons.org/licenses/by/3.0/).  
Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

## 1. Introduction

When a container partly filled with granular material is agitated by an oscillating spring, energy of the spring is transferred to the particles and dissipated due to inelastic collisions of the particles with one another and with the container, resulting in attenuation of the amplitude of the oscillation. This is the basic principle of granular dampers which are used in a number of technical applications, such as mechanical tools and machinery [1–6], metal cutting machines [7], turbine and compressor blisks [8, 9], sports equipment [10, 11], vibrating antennae [12–14], bonding machines [15] and had also been proposed to reduce vibrations of the space shuttle engine [16, 17].

While the amplitude of a linearly damped harmonic oscillator decays exponentially in time, the amplitude of a vibrating spring with an attached granular damper decays almost linearly in time up to a certain value. From there on, the amplitude decays much slower. This linear decay was reported in *many* references based on both experimental work, e.g. [15, 18–22], and particle simulations, e.g. [23–26]. Similar results apply also to impact dampers, e.g. [27–30].

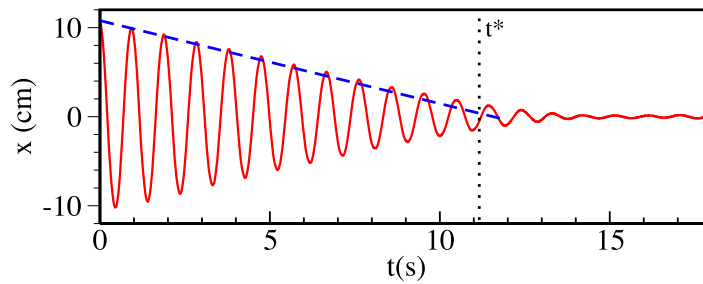
The aim of this paper is to explain the two regimes of granular damping of a spring, that is, linear attenuation of the amplitude followed by a period of weaker damping which is a long standing problem. We will show that this behavior may be attributed to different collective modes of dynamics of granular matter in an oscillating box. For our investigation we wish to exclude the influence of gravity on the dynamics of excited granular matter and, thus, on granular damping [21, 31–36]. Therefore, we refer to recent experiments on granular damping under conditions of microgravity [37].

## 2. Linear decay of the amplitude

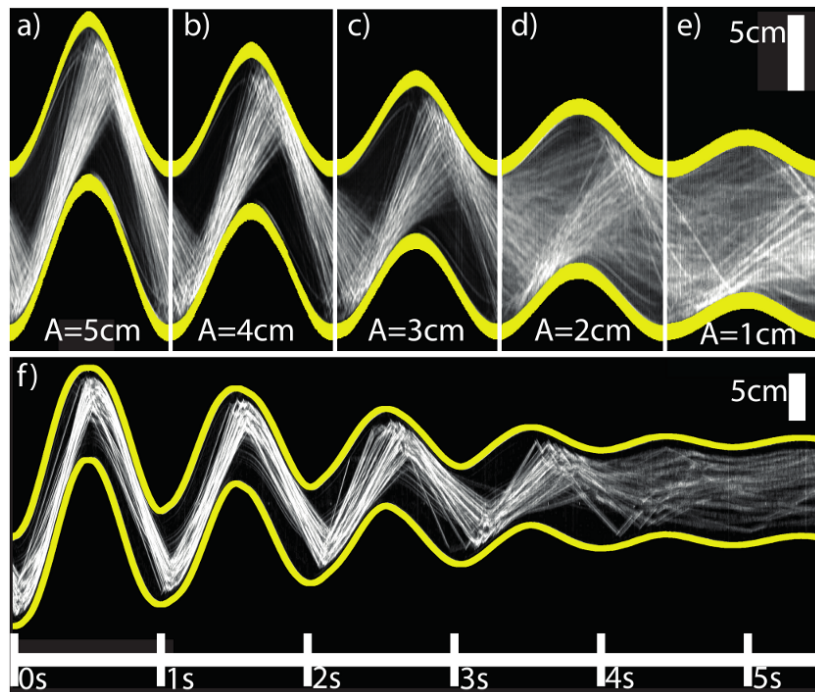
Figure 1 shows the decay of the amplitude of a spring's oscillation with an attached granular damper. Similar figures can be found in many publications on granular damping [15, 18–30] which have three characteristic features in common: (a) for large amplitude, the decay follows an almost linear function of time; (b) for small amplitude (large time) the decay is weak and (c) there is a rather sharp transition between both regimes. In our case, the figure shows the attenuation of a flat spring where the granular damper is a rectangular polycarbonate container of mass  $M = 434$  g and length  $L = 4$  cm in the direction of vibration and cross section  $5 \times 5$  cm<sup>2</sup>, mounted on top of a flat spring ( $k = 24.4$  Nm<sup>-1</sup>) and loaded with 37 steel balls (diameter 1 cm, total mass  $m = 149$  g). In its initial position, the container is deflected by about  $x(t = 0) = A_0 = 11$  cm. The time dependent position,  $x(t)$ , was recorded by means of a high-speed camera. The experiment was performed during a parabolic flight under conditions of microgravity, for details see [38].

## 3. Modes of excitation

From the high-speed video recording, we notice a transition of the dynamical behavior of the granulate at the same time,  $t^*$ , when the mode of the attenuation changes. To describe the dynamical modes we first look at an externally driven system, that is, the container follows a sinusoidal motion at fixed amplitude and frequency, figures 2(a)–(e). To obtain each subfigure of figure 2 we computed for each frame of the high-speed recording the average gray value in the plane perpendicular to the oscillation. This way, each frame was condensed to a single line.



**Figure 1.** Typical attenuation of a linear spring damped by a granular damper [38]. Two regimes are separated by a rather sharp transition at time  $t^* \approx 11$  s. The linear decay (dashed line) for  $t < t^*$  was reported in many publications, e.g. [15, 18–30].



**Figure 2.** Modes of excitation of the granulate in a vibrating container, obtained from the high-speed recording of the oscillating box during one period (see text). (a)–(e) Each sub-figure shows the granulate moving in a box sinusoidally driven at constant amplitude  $A$ . (f) The granulate moves in a box attached to an oscillating spring for time  $t = 0–5$  s. The corresponding decay of the amplitude over time is shown in figure 3(c). For better visibility, the position of the container (also obtained from the video data) is highlighted in color.

These lines were stacked-up to give an image of the flow of granulate during the oscillatory motion. For large amplitude,  $A(t)$ , see figures 2(a)–(c), the particles move collectively as a cluster and arrive at the wall at a phase of oscillation when the wall is accelerating inwards, i.e. toward the colliding particles. This way, the particles arriving at the incoming wall are *collected*,

and accelerated in a compact state toward the opposite wall where they arrive again collectively. This mode of behavior was termed *collect-and-collide* regime [38].

For small amplitude, figures 2(d) and (e), we do not observe collective motion of the granulate. In this regime, the granulate adopts a gaseous state, occupying the entire volume of the container nearly homogeneously. Note that these two states, collect-and-collide and gaseous, are also found in numerical simulations [35] as extremal states of granular dynamics. It was shown recently [37] that the loss of mechanical energy due to dissipative particle collisions is fundamentally different in both regimes of dynamical behavior.

Comparing the dynamics of the granulate in the driven system at fixed amplitude, figures 2(a)–(e), with the dynamics in a container attached to a spring where the amplitude is a decaying function of time, figure 2(f), we observe very similar modes. For large amplitude,  $A > A^*$  (corresponding to  $t < t^*$ ) the granulate shows collect-and-collide dynamics; for  $A < A^*$  (corresponding to  $t > t^*$ ) it behaves like a granular gas. This similarity is not trivial since *a priori* it cannot be excluded that there is a long lasting transient state separating the collect-and-collide regime from the gaseous behavior. Comparing figures 2(a)–(e) with figure 2(f), we conclude that the transition time,  $t^*$ , identified in figure 1 as the time when the relaxation process changes its mode, corresponds to the transition of the granulate's dynamical behavior in the box from the collect-and-collide regime to a gaseous state.

#### 4. Linear relaxation of a damped spring

The similarity of the granulate's dynamical modes for the cases of external agitation at constant amplitude and with decaying amplitude suggests to consider the relaxation as a sequence of steady-states. This is certainly justified if the relaxation time is much larger than the period of the oscillation. We will see below that the results obtained under this conditions stay approximately correct also beyond this limit.

Initially, the total energy of the system is  $E_0 = kA_0^2/2$ , where  $k$  is the elastic constant of the driving spring and  $A_0$  is the initial elongation of the oscillator from its equilibrium position,  $A_0 = x(t = 0)$ , see figure 1. Following the arguments above, in the collect-and-collide regime, the granular material behaves essentially like a single particle which collides perfectly inelastically with the container wall. That is, twice per period, the granulate collides with the wall and loses its velocity relative to the container. The corresponding loss of mechanical energy leads to the attenuation of the amplitude  $A(t)$ .

To quantify  $A(t)$ , we look at the dynamics of the system during one half period in the collect-and-collide regime, starting at  $x = 0$ ,  $\dot{x} > 0$  which we attribute to the time  $\tau = 0$ . Consider first the case of external sinusoidal driving at constant amplitude,  $x(\tau) = A \sin(\omega\tau)$ . At time  $\tau = 0$ , that is,  $x = 0$ , the container travels at maximum velocity  $V_{\max} = A\omega$ . At this time, the granulate is accumulated at the back wall of the container due to the preceding inward stroke and moves at velocity  $v = V_{\max}$  synchronously to the container. At this point,  $x = 0$ , the granulate decouples from the container. The container is decelerated,  $V = A\omega \cos(\omega\tau)$ , while the granulate continues moving at velocity  $v$ . The granulate collides with the opposite wall of the container at time  $\tau_c$  after the container reached the maximal elongation,  $x = A$  at time  $\tau = \pi/2\omega$ , (see figure 2(b)) being a condition for the stability of the collect-and-collide regime, see [37] for a detailed discussion. The time of collision,  $\tau_c$ , is obtained from the distance traveled by the granulate at velocity  $v$  and the harmonic motion of the container:

$$v\tau_c = A\omega\tau_c = A \sin(\omega\tau_c) + L_g, \quad (1)$$

where the clearance,  $L_g$ , is the difference between box length and the thickness of the packed layer of particles in the box. That is,  $L_g$  is the distance traveled by the granulate during one stroke, relative to the container. The velocity of the box at the time of impact is then given by  $V_c = A\omega \cos(\omega\tau_c)$ .

Let us now discuss the case of the oscillator driven by a spring. Here,  $\omega$  is the native frequency of the spring with the attached container. We checked that (a) during the entire process of attenuation the shape of the oscillation is almost perfectly sinusoidal,  $x(t) = A(t) \sin(\omega t)$ , and (b) the frequency,  $\omega$ , is independent of the amplitude. This observation allows to consider the frequency as a system parameter. In the experiments discussed here, we determined  $\omega = 2\pi \times 1.05 \text{ s}^{-1}$ .

In order to derive an equation for the attenuation,  $A(t)$ , first we note that during the entire interval,  $\tau \in (0, \tau_c)$ , the container moves decoupled from the granulate, therefore, we equate the kinetic energy of the container of mass  $M$  at  $x = 0$  and the potential energy of the spring at maximal elongation at  $x = A$ :

$$\frac{k}{2}A^2 = \frac{M}{2}v^2 \quad (2)$$

to obtain the velocity of the granulate when it decouples from the container,  $v = A\sqrt{k/M}$ .

When the container and the granulate of total mass  $m$  collide at velocities  $V_c$  and  $v$  perfectly dissipatively such that the postcollisional relative velocity vanishes, the dissipated energy is

$$\begin{aligned} E_{\text{diss}} &= -\frac{1}{2}m_{\text{eff}}(V_c - v)^2 \quad \text{with} \quad m_{\text{eff}} = \frac{mM}{m+M} \\ &= -\frac{1}{2}m_{\text{eff}} \left[ A\omega \cos(\omega\tau_c) - A\sqrt{\frac{k}{M}} \right]^2. \end{aligned} \quad (3)$$

The dissipation of energy leads to attenuation of the amplitude  $A$  of the container's oscillation

$$E_{\text{diss}} = \frac{k}{2} \left[ A^2 \left( t + \frac{T}{2} \right) - A^2(t) \right], \quad (4)$$

where the argument  $t + T/2$  takes into account that the energy  $E_{\text{diss}}$  is dissipated in half of the period of oscillation,  $T = 2\pi/\omega$ . With the assumption that  $T/2$  is small as compared to the characteristic relaxation time of the oscillation, we write

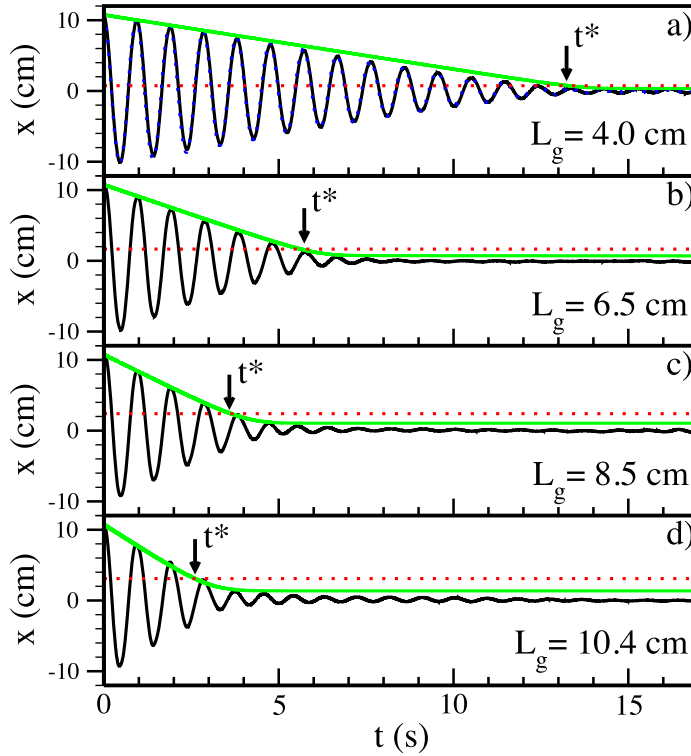
$$E_{\text{diss}} \approx \frac{k\pi}{\omega} A(t) \frac{dA}{dt}. \quad (5)$$

From equations (3) and (5) we obtain an equation for the attenuation of the amplitude,  $A(t)$ :

$$\gamma \frac{dA}{dt} = -A(t) \frac{\omega m_{\text{eff}}}{2\pi k} \left[ \omega \cos(\omega\tau_c) - \sqrt{\frac{k}{M}} \right]^2 \quad (6)$$

with the initial elongation  $A(t=0) = A_0$ . The amplitude  $A$  enters the rhs via  $\tau_c$  since the argument of the cosine function is the solution of

$$\omega\tau_c = \sin(\omega\tau_c) + L_g/A. \quad (7)$$



**Figure 3.** Relaxation of an oscillating spring with attached granular damper for different values of the clearance,  $L_g$ . The green lines show the amplitude,  $A(t)$ , obtained from the numerical solution of equation (6). The dashed lines indicate the residual amplitude,  $A^r(L_g)$ , given by equation (9). The time of transition,  $t^*$ , follows from the initial slope,  $-\frac{dA}{dt}|_{t \rightarrow 0}$  and  $A^r$ .

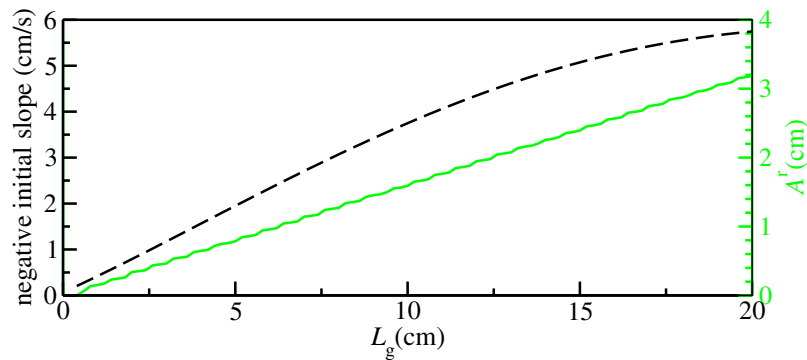
The factor  $\gamma$  was introduced in the lhs of equation (6) to compensate for additional losses of energy due to the inherent damping of the driving spring. For our system, we found the best agreement between the experiment and the numerical solution of equation (6) for the value  $\gamma = 0.85$ . Figure 3 shows the oscillation of the damper found in experiments together with the numerical solution of equation (6) for different values of the clearance,  $L_g$ .

The residual amplitude,  $A^r$ , corresponds to the transition from the collect-and-collide mode into the gas regime: a condition for the collect-and-collide regime is that the incoming particles meet the wall when it moves accelerated *toward* them [37]. This is the case for  $\omega\tau_c < \pi$ . Otherwise, the particles cannot be collected but are immediately scattered back when they (individually) arrive —collect-and-collide is not possible, see figure 2(f) at  $t \approx 4$  s. From the first order expansion of equation (7) around  $\omega\tau_c = \pi$  we obtain

$$\omega\tau_c = \frac{\pi}{2} + \frac{L_g}{2A}, \quad (8)$$

which relates the condition  $\omega\tau_c < \pi$  to the residual amplitude

$$A^r = \frac{L_g}{\pi} \quad (9)$$



**Figure 4.** Residual amplitude,  $A^r$ , as a function of the clearance,  $L_g$ , obtained from the solution of equation (6) (full line). The efficiency of damping is characterized by the initial slope of the amplitude,  $-\frac{dA}{dt}\big|_{t \rightarrow 0}$  (dashed line).

where the collect-and-collide regime ceases. The dotted line in figure 3 shows  $A^r$  due to equation (9) for our experiments. In all cases, the theoretical value of  $A^r$  appears to be too pessimistic, that is, the model description stays valid beyond the limit given by equation (9).

Obviously, the initial slope of the almost linear decay of  $A(t)$ , characterizing the efficiency of damping, increases with  $L_g$ . At the same time, the residual amplitude,  $A^r$ , increases with  $L_g$  as well. Consequently, when applying granular dampers, one has to compromise between the aims of small residual amplitude and efficiency of damping. Figure 4 shows  $A^r(L_g)$  due to equation (9) together with  $A^r$  obtained from the numerical solution of equation (6) taken at the time  $t^*$  when the collect-and-collide regime ceases. The efficiency of damping is shown in figure 4 expressed by the initial slope of the amplitude,  $-\frac{dA}{dt}\big|_{t \rightarrow 0}$ .

## 5. Conclusion

When an oscillating spring is attenuated by a granular damper, initially the amplitude of the oscillation decays apparently linearly in time. When a certain value of the amplitude is reached, the linear decay ceases and a much weaker decay is found. This behavior was reported in *many* references, but the physical reason for this untypical damping behavior remained obscure. It is the aim of this paper to give a quantitative explanation for the initial linear decay and the residual amplitude when the linear decay ceases which are long-standing questions.

By means of experiments regarding both externally driven granular systems and relaxation boxes filled with granulate attached to an oscillating spring, both performed under conditions of microgravity, we found that the transition of the damping behavior is attributed to a transition of the dynamical state of the granulate: for large amplitude,  $A > A^*$ , the particles move collectively as a cluster whose center of mass follows the oscillation of the spring. This type of dynamical behavior was termed *collect-and-collide regime*. For small amplitude,  $A < A^*$ , the material adopts a gaseous state.

In the collect-and-collide regime, the dissipated energy may be quantified using an effective one-particle model. We apply this result to the relaxation process of a spring damped by a granular damper, to obtain an equation for the attenuation of the amplitude,  $A(t)$ , whose numerical solution agrees well with the observed (but yet unexplained) linear decay. To consider

the relaxation process as a sequence of stationary states may appear questionable since *à priori* we cannot exclude long-lasting transient states. However, the comparison of the theoretical result with experiments yields quantitative agreement justifying the assumption. Besides the slope of the initial decay, the model also predicts the transition from the collect-and-collide regime to the gas state and the residual amplitude,  $A^r$ , in quantitative agreement with the experiment.

The initial slope of the amplitude,  $\dot{A}(0)$ , and the residual amplitude,  $A^r$ , when the (rapid) linear decay ceases, are both increasing functions of the clearance  $L_g$  determined by the filling ratio of the damper. Therefore, when applying granular dampers, one has to compromise between efficient damping and final amplitude. Thus, in practice, a combination of several granular dampers representing different combinations of  $[\dot{A}(0), A^r]$  may be favorable.

Finally, we wish to mention that the initial decay follows only approximately a linear decay, observed in many experiments and simulations. In the strict sense, the decay is described by a nonlinear function, however, the nonlinearity is rather weak such that in many experiments the decay appears linear.

## Acknowledgments

We thank the German Science Foundation (DFG) for funding through the Cluster of Excellence ‘Engineering of Advanced Materials’ as well as the German Aerospace Center (DLR) and the European Space Agency (ESA) for funding the parabolic flight campaigns. We further acknowledge additional support by DFG and Friedrich-Alexander-Universität Erlangen-Nürnberg within the funding programme Open Access Publishing.

## References

- [1] Norcross J C 1967 Dead-blow hammer head *US Patent* 3343576
- [2] Erlikh L B 1953 *Eng. Dig.* **14** 31
- [3] Sadek M M 1972 *Mach. Prod. Eng.* **120** 152
- [4] Sadek M M and Mills B 1966 *Proc. 7th Int. Machine Tool Conf. (Birmingham)* pp 243–57
- [5] Sims N D, Amarasinghe A and Ridgway K 2005 *Manuf. Eng.* **16-1** 825–32
- [6] Heckel M, Sack A, Kollmer J E and Pöschel T 2012 *Physica A* **391** 4442–7
- [7] Ryzhkov D I 1953 *Eng. Dig.* **14** 246
- [8] Paget A L 1937 *Engineering* **143** 305–7
- [9] Kielb R, Macri F, Oeth D, Nashif A, Macioce P, Panossian H and Lieghley F 1999 Advanced damping systems for fan and compressor blisks *Proc. 4th National Turbine Engine High Cycle Fatigue Conf.* (Monterey, CA)
- [10] Sommer R 1995 Sports equipment for ball games having an improved attenuation of oscillations and kick-back pulses and an increased striking force *US Patent* 5454562
- [11] Ashley S 1995 *Mech. Eng.* **117** 80–1
- [12] Rocke R D and Masri S F 1969 *Shock Vib. Bull.* **39** 1–10
- [13] Langer W and Strienz G 1991 *IfL-Mitt.* **30** 46–53
- [14] Simonian S S 1995 *Proc. SPIE* **2445** 149–60
- [15] Chan K W, Liao W H, Wang M Y and Choy P K 2006 *J. Vib. Control* **12** 297–312
- [16] Panossian H V 1992 *ASME J. Vib. Acoust.* **114** 101–5
- [17] Simonian S S 2004 *Collection of Technical Papers—AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conf.* **6** 4145–61
- [18] Friend R D and Kinra V K 1999 *Proc. SPIE* **3672** 20–31



- [19] Marhadi K S and Kinra V K 2005 *J. Sound Vib.* **283** 433–48
- [20] Xu Z W, Wang M Y and Chen T N 2004 *J. Vib. Acoust.* **126** 141–8
- [21] Bai X M, Shah B, Keer L M, Wang Q J and Snurr R Q 2009 *Powder Technol.* **189** 115–25
- [22] Shah B M, Pillet D, Bai X M, Keer L M, Wang Q J and Snurr R Q 2009 *J. Sound Vib.* **326** 489–502
- [23] Fang X and Tang J 2006 *J. Vib. Acoust.* **128** 489–500
- [24] Mao K, Wang M Y, Xu Z and Chen T 2004 *Powder Technol.* **142** 154–65
- [25] Mao K, Wang M Y, Xu Z and Chan T 2004 *J. Vib. Acoust.* **126** 202–11
- [26] Bai X M, Keer L M, Wang Q J and Snurr R Q 2009 *Gran. Mater.* **11** 417–29
- [27] Bapat C N and Sankar S 1985 *J. Sound Vib.* **99** 85–94
- [28] Ekwaro-Osire S and Desen I C 2001 *J. Vib. Control* **7** 475–93
- [29] Cheng J and Xu H 2006 *Int. J. Solids Struct.* **43** 5355–69
- [30] Vinayaravi R, Kumaresan D, Jayaraj K, Asraff A K and Muthukumar R 2013 *J. Sound Vib.* **332** 1324–34
- [31] Sadek M M and Mills B 1970 *J. Mech. Eng. Sci.* **12** 268–77
- [32] Sadek M M and Williams C J H 1970 *J. Mech. Eng. Sci.* **12** 278–87
- [33] Salueña C, Pöschel T and Esipov S E 1999 *Phys. Rev. E* **59** 4422–5
- [34] Salueña C, Esipov S E, Pöschel T and Simonian S 1998 *Proc. SPIE* **3327** 19–26
- [35] Opsomer E, Ludewig F and Vandewalle N 2011 *Phys. Rev. E* **84** 051306
- [36] Sánchez M, Rosenthal G and Pugnaroni L A 2012 *J. Sound Vib.* **331** 4389–94
- [37] Sack A, Heckel M, Kollmer J E, Zimmer F and Pöschel T 2013 *Phys. Rev. Lett.* **111** 018001
- [38] Bannerman M N, Kollmer J E, Sack A, Heckel M, Müller P and Pöschel T 2011 *Phys. Rev. E* **84** 011301