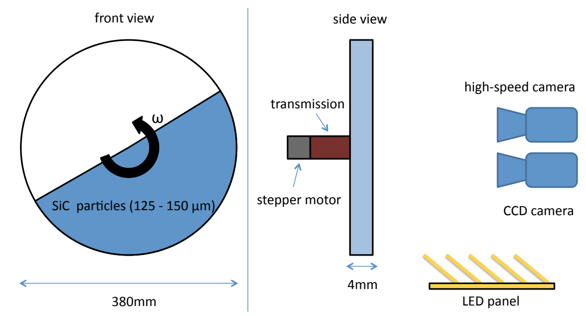
Friedrich-Alexander-Universität Erlangen-Nürnberg
ENGINEERING OF ADVANCED MATERIALS
Institute for Multiscale Simulation

Poly-directional Stability of Granular Matter

F. Zimber, J. E. Kollmer and T. Pöschel

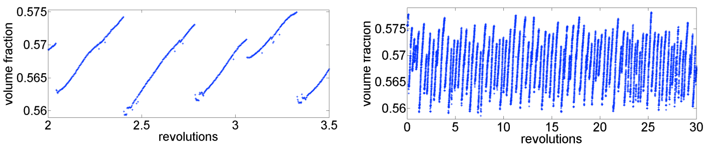
By means of a rotating drum experiment, we introduce the state of *poly-directional stability* in jammed granular matter where the material responds elastically to small stresses in a wide angular interval. Only to small stresses which are directed in a relatively small interval of directions the material responds by plastic deformations. The state of poly-directional stability complements the *fragile* [1] state, where the material responds elastically to small applied stresses only in a certain direction but even very small stress in any other direction would lead to plastic deformations. Similar to fragile matter, poly-directionally stable matter is created in a dynamic process by self-organization.

Rotating Drum Experiment

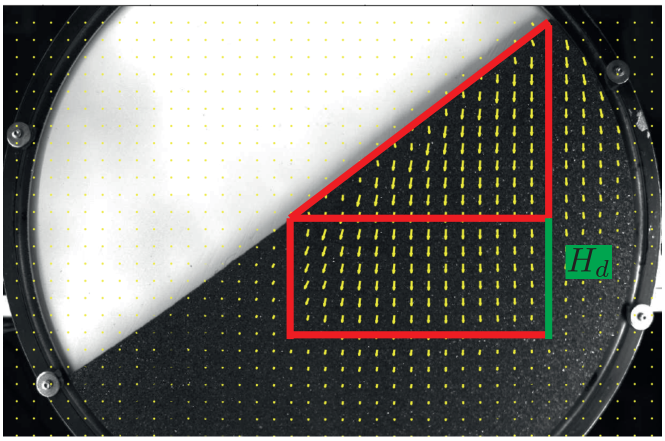


Recurrent Inflation & Sudden Collapse

Slow rotation (1/7 RPM) creates a stationary state that is characterized by a steady narrow flow (no stick-slip) of grains downhill the free surface interrupted by sudden collapses of the sediment.

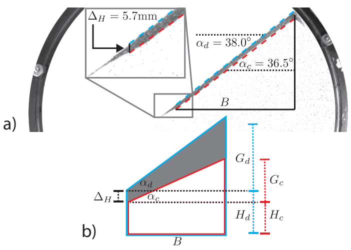


Volume fraction as a function of time in the steady state. Left: the total volume of the material varies by about 2% due to inflation in the steady flow regime and collapses. Right: the same date for a shorter interval showing the inflation as a linear function of time.



Cumulative velocity field during a collapse obtained by PIV. The vector arrows are magnified for better visibility. The region of non-vanishing velocities indicate the volume affected by the

From the trapezoidal shape of the collapsed volume we can compute the density ratio of dilute and collapsed material:



Overlay of the images taken immediately before and after a collapse. The image prior to the collapse is drawn in reverse gray scale, α_d and α_c indicate the slope of the surface before and after the collapse.

The mass of low density material located in the triangle △(B,G_d) collapses into the triangle △(B,G_c) and thus

$$\frac{\rho_{\text{collapsed}}}{\rho_{\text{dilute}}} = \frac{\tan\alpha_d}{\tan\alpha_c} = 1.053$$

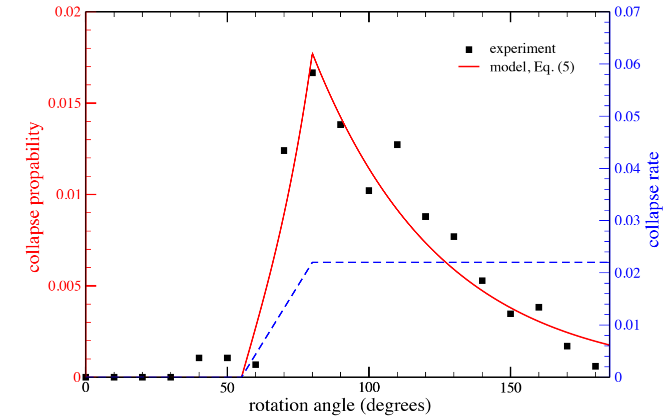
Similarly, the material contained in the rectangle □(B,H_d) collapses into □(B,H_c) which gives

$$H_d = \frac{\Delta H}{1 - \frac{\rho_{\text{dilute}}}{\rho_{\text{collapsed}}}} = 102\text{mm}$$

and agrees well with the PIV result.

Rate Model

Generally we observe that collapse events are always restricted to the right side of the cylinder, thus material rotated by less than 90°- φ_R is always stable. We analyze the series of collapses with respect to the distances between consecutive collapse events and obtain the histogram below:

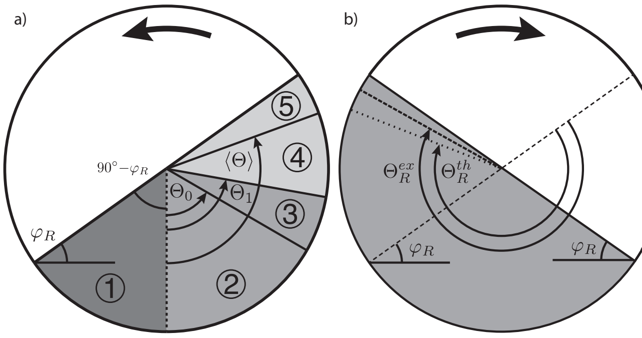


Normalized frequencies of intervals between consecutive collapse events found in the experiment (points). Almost no collapses occur until the cylinder is rotated by about 60° after the preceding collapse. The solid line show the probability density based on the rate model. The rate is sketched by the dotted line (right axis).

In the interval 0 ≤ Θ ≤ 60° after a collapse event we find almost no collapses, followed by a peak at Θ ≈ 80°. This means that the lose material is stable in the corresponding orientation. We model the system's behavior by a rate model assuming that for $\Theta < \Theta_0$ the dilute material is stable and insensitive to small perturbations which are always present when the cylinder is rotated. For $\Theta > \Theta_1$ the material is oriented such that it left its angular range of stability and even a small perturbation may cause a collapse.

$$r_c(\Theta) = \begin{cases} 0 & \text{for } 0 \le \Theta \le \Theta_0 \\ r_0 \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0} & \text{for } \Theta_0 \le \Theta \le \Theta_1 \\ r_0 & \text{for } \Theta \ge \Theta_1 \end{cases}$$

$$p(\Theta) = \begin{cases} 0 & \text{for } \Theta < \Theta_0 \\ r_0 \exp\left(\frac{r_0}{2}\frac{(\Theta-\Theta_0)^2}{\Theta_1-\Theta_0}\right)\frac{\Theta-\Theta_0}{\Theta_1-\Theta_0} & \text{for } \Theta_0 \le \Theta < \Theta_1 \\ r_0 \exp\left(\frac{r_0}{2}(3\Theta_1 - \Theta_0)\right) & \text{for } \Theta \ge \Theta_1 \end{cases}$$



Reversal of Rotiation & Polydirectional Stability

Reversal of the rotation. To confirm our hypothesis on the wide angle of structural stability we reverted the sense of the rotation in the following way: We waited for a collapse-free interval long enough that we can expect that all material in the container is in the dilute state. At this point ($\Theta_r = 0$) we restarted the rotation in opposite direction. The surface flow restarted in the opposite direction when the material surface reached again the angle of repose, and the first collapse event was observed at $\Theta_r \approx -225.7°$. Consequently, in this experiment we found a new state of dilute jammed granular matter which is stable against small perturbation in a wide angular interval of small stresses whereas it responds plastically when loaded with stresses outside this interval.

[1] M.E.Cates, J.P.Wittmer, J.-P.Bouchaud, and P.Claudin, Phys. Rev. Lett. 81, 1841 (1998)