The coefficient of normal restitution as a fluctuating quantity

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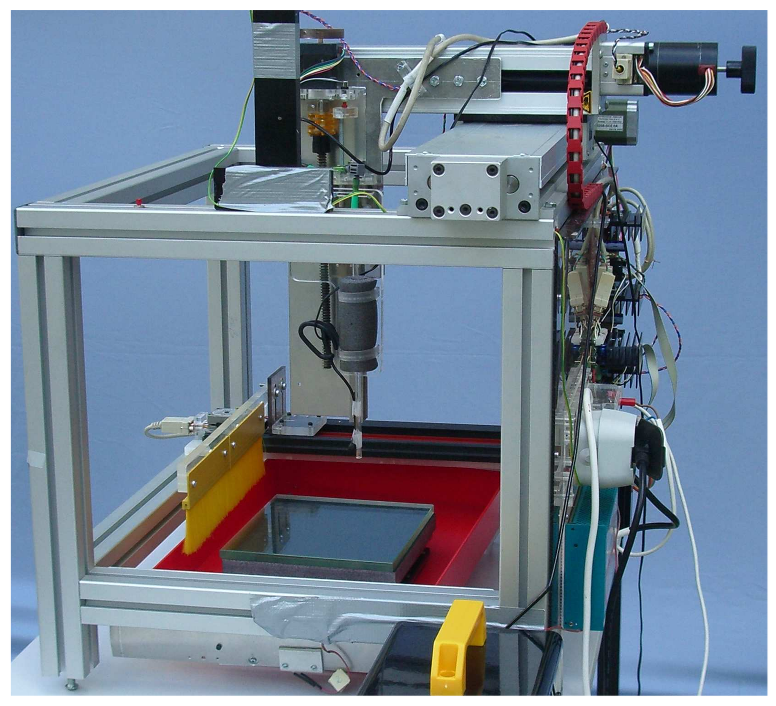
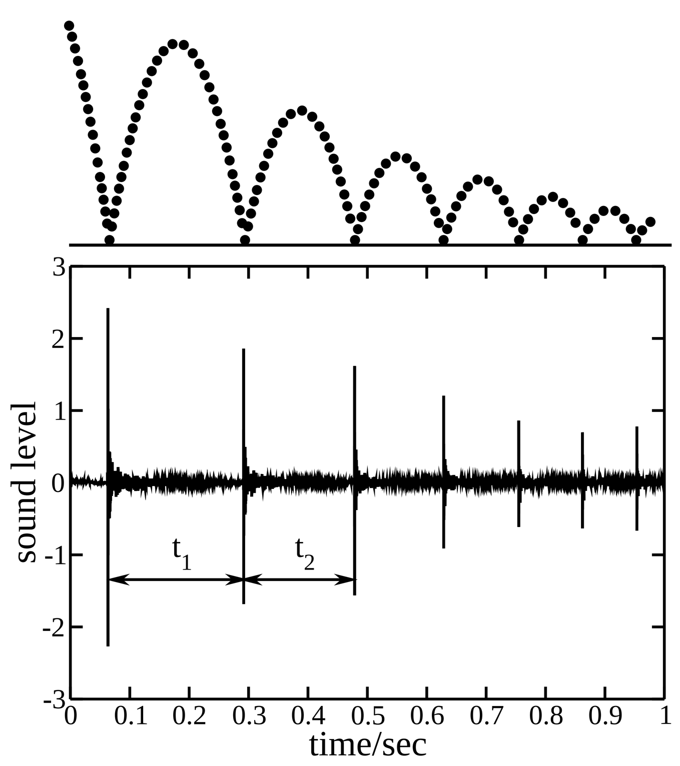
Introduction

Granular particles lose a certain part of their kinetic energy in dissipative collisions. The degree of energy loss, i.e. their dissipative properties are characterized by the coefficient of restitution. It is defined as the ratio between the post-collision and the pre-collision velocities. One defines two separate coefficients of restitution for the normal and for the tangential component of the relative velocity:

$$\varepsilon_n = -\frac{g_n'}{g_n} \qquad 0 \le \varepsilon_n \le 1$$
$$\varepsilon_t = \frac{g_t'}{g_t} \qquad -1 \le \varepsilon_t \le 1$$

The most simple experiment to measure the coefficient of normal restitution is to drop a sphere from a certain height and listen to the sound of the repeated impacts of the sphere [1-3]. Using simply a microphone and recording the sound with the sound-card of a desktop PC one can accurately measure the times of impact.

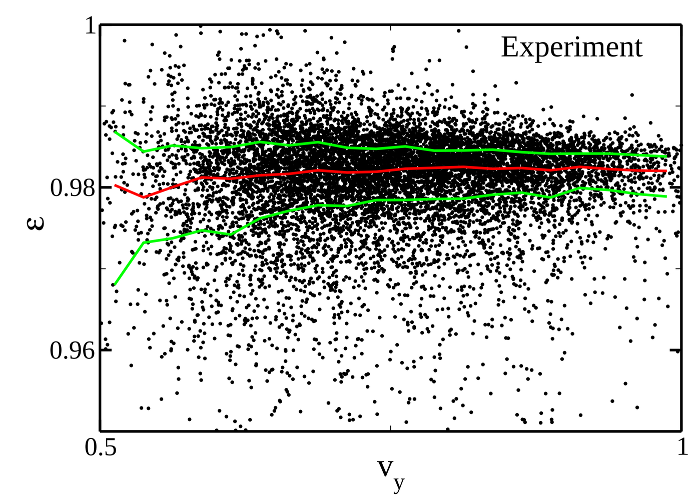




The velocity of the particle can be deduced from the time interval between two impacts. If t_1 and t_2 are the time intervals before and after a certain collision one can compute the pre-collision velocity g_n, the post-collision velocity g_n' and the coefficient of normal restitution (with G being Earth's gravity):

$$g_n = -\frac{Gt_1}{2}$$
$$g_n' = \frac{Gt_2}{2}$$
$$\varepsilon_n(g_n) = \frac{t_2}{t_1}$$

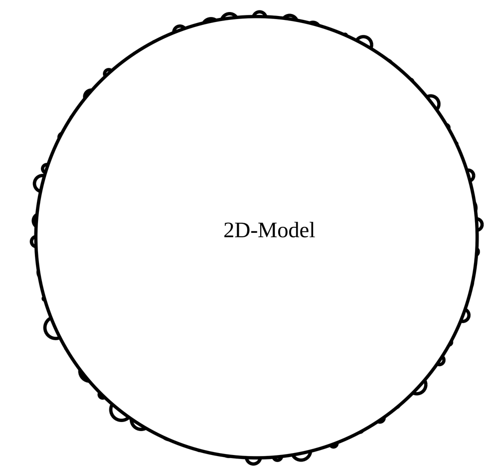
Repeating this simple experiment for many times one notes that the coefficient of restitution does not seem to be a reproducible quantity. Instead it is subject to a considerable amount of noise.

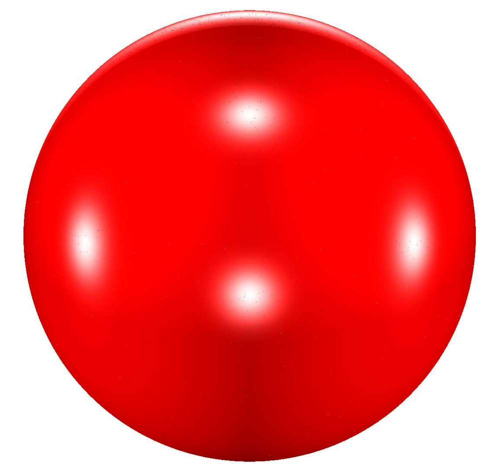


The red curve is the median (for a given velocity), the green lines contain the 25% above and below the median. The figure shows 10 000 data points.

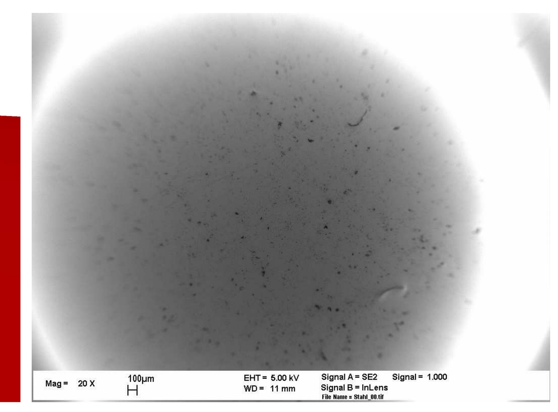
Particle model

We assume that the large data noise is caused by the surface roughness of the particles. To model the suspected surface features we constructed a complicated particle consisting of a large central sphere of radius R and many (a few thousands) of small ($0.001R$) spheres which represent the asperities. They are fixed to the surface of the central particle. From a distance the particle appears as a perfect sphere:



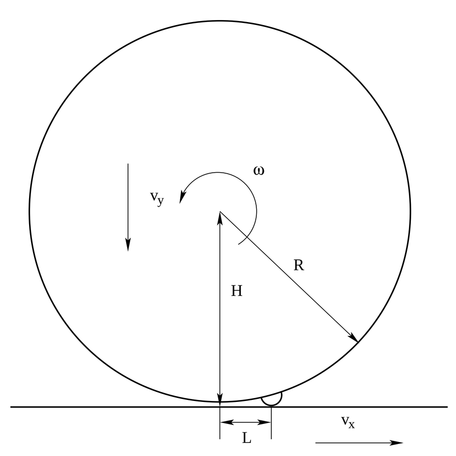


A close-up reveals the asperities (left panel below). Electron-microscope pictures of the spheres used in the experiment reveal similar features. (right panel below).



Collision Mechanics

The asperities provide a lever yielding a non-central collision. Asperities of elevation h provide a typical lever length of $L_{\max} = \sqrt{2Rh}$. The particle is treated as perfectly rigid. During the collision the contact impuls $\Delta\mathbf{P}$ is transferred to the particle. This results in a change of velocity and angular velocity. In 2d we have



$$v_x' = v_x + \frac{\Delta P_x}{m}$$
$$v_y' = v_y + \frac{\Delta P_y}{m}$$
$$\omega' = \omega + \frac{H\Delta P_x + L\Delta P_y}{J}$$

Together with the definition of the normal velocity components

$$g_t = v_x + H\omega$$
$$g_n = v_y + L\omega$$

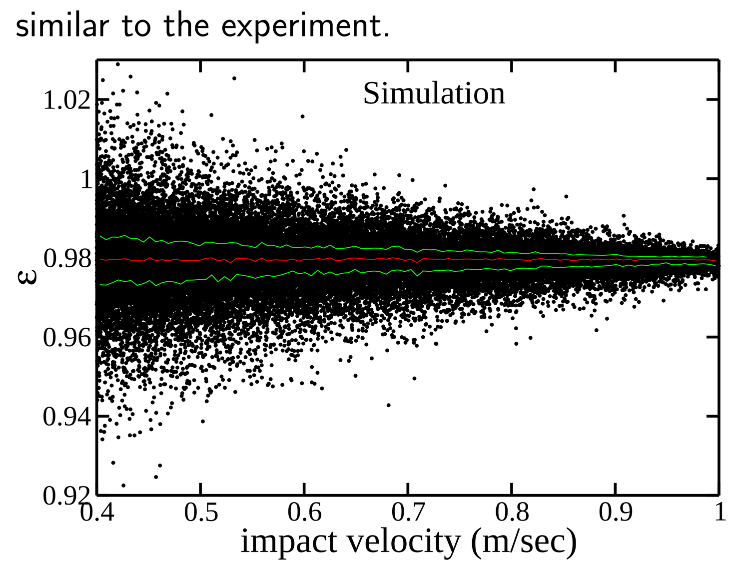
and the definition of the coefficients of restitution we obtain the necessary values of ΔP_x and ΔP_y which in turn yields the post-collisional values of v_x, v_y and ω. As the relative velocity of the point of contact is unknown the coefficient of restitution in the experiment is not $-g_n'/g_n$ but the approximation $-v_y'/v_y$. In good approximation we have

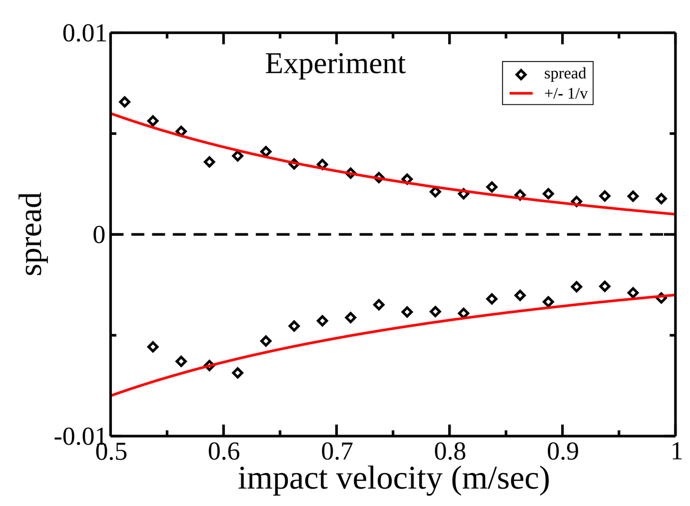
$$\varepsilon_{\exp} = \varepsilon_n + (1+\varepsilon_n)\frac{\text{random}}{v_y} \qquad (1)$$

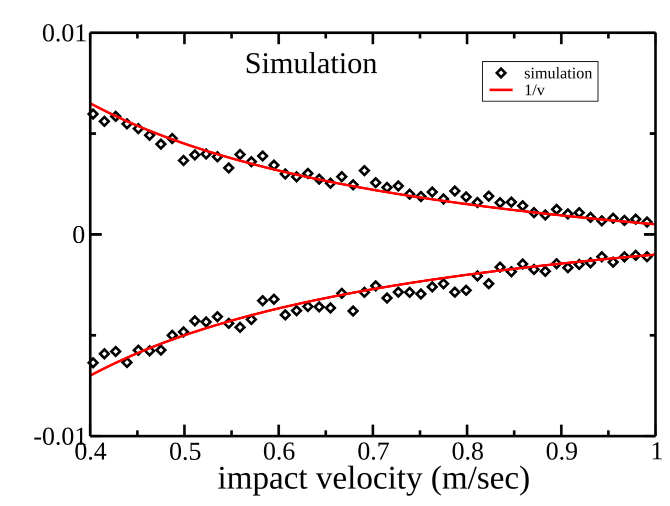
The random number drawn from the interval $[-L_{\max}..L_{\max}]$. From the collision mechanics we conclude that the noise increases as $1/v_y$.

Results

Simulating the experiment with the model particles one obtains coefficients of restitution very similar to the experiment.

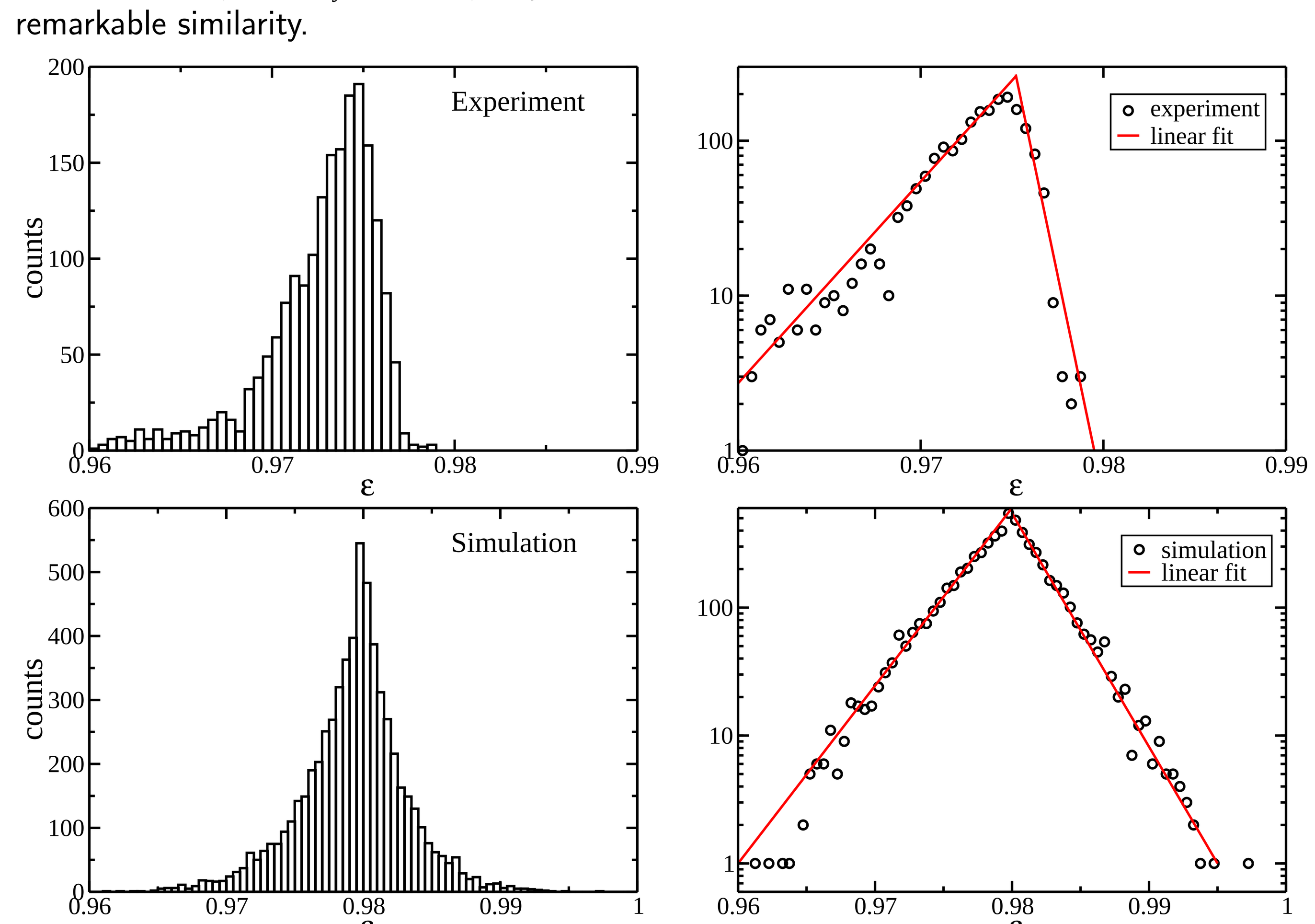






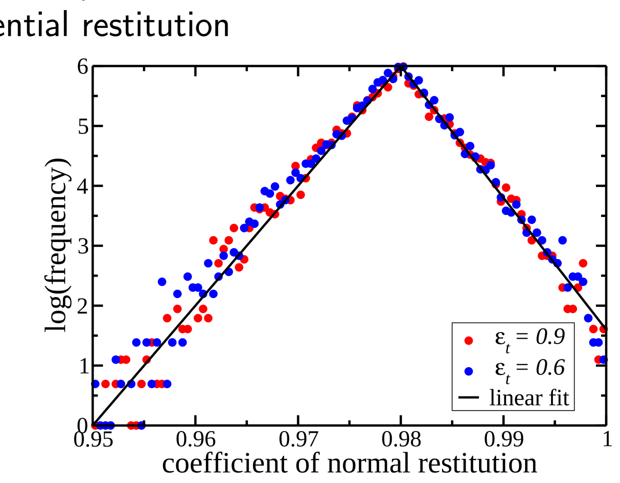
Specifically, for both simulation and experiment the spread of the data (the interval enclosing the 25% above and below the median) opens as $1/v_y$.

Selecting all coefficients of restitution from a narrow velocity interval (e.g. all ε for impact velocity 0.75m/sec$\le v_y \le$0.85m/sec) and plotting the distribution of the ε one observes a remarkable similarity.



In both the experiment as in the simulation the distribution is asymmetric. The log-plot reveals that the distribution is exponential in both cases. This astonishing fact is as of now unexplained and subject of current research.

Eq. (1) predicts that the *measured* coefficient of restitution is practically independent of the *intrinsic* coefficient of tangential restitution. Plotting the distribution of ε (for a narrow velocity interval) for different intrinsic coefficients of tangential restitution



one observes no visible dependence on the tangential restitution.

The coefficient of restitution is an intrinsically stochastic quantity due to surface impurities. The fluctuations are non-Gaussian. A simple particle model with asperities is sufficient to explain the experimental measurements. In future studies the stochastic coefficient of restitution will be applied to investigate the kinetic properties of granular gases.

References

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2 P.A. Smith, C.D. Spencer, and D.E. Jones (1981), *Am. J. Phys. 49*, 136.
3 I. Stensgaard and E. Lægsgaard (2001), *Am. J. Phys. 69*, 301.

Upcoming publication:
T. Schwager, C. Krülle, and T. Pöschel, "The coefficient of restitution: What can we learn from hopping ball experiments", *in preparation*