Does the oscillation of a sphere affect its coefficient of restitution?

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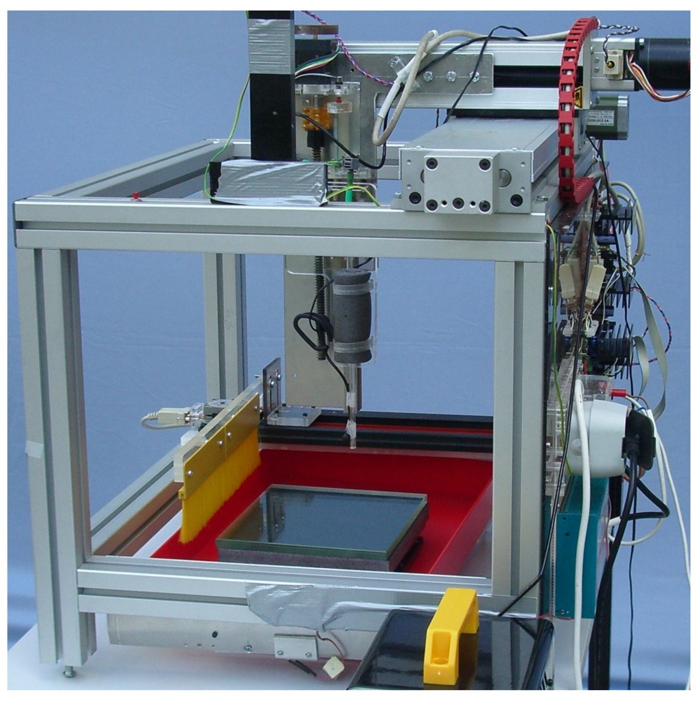
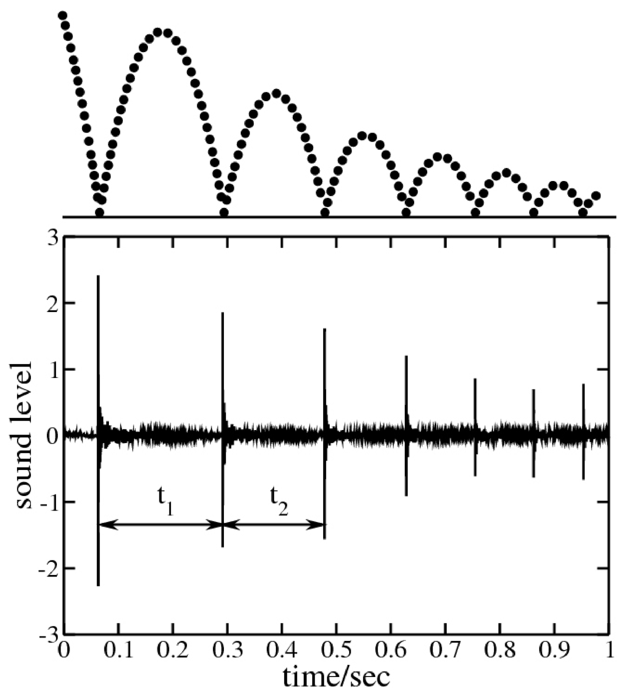
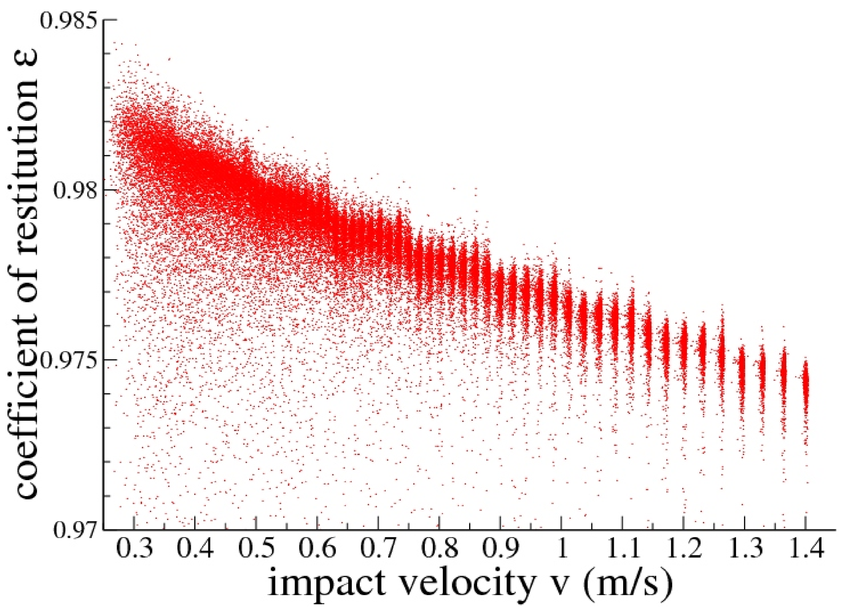
Setup

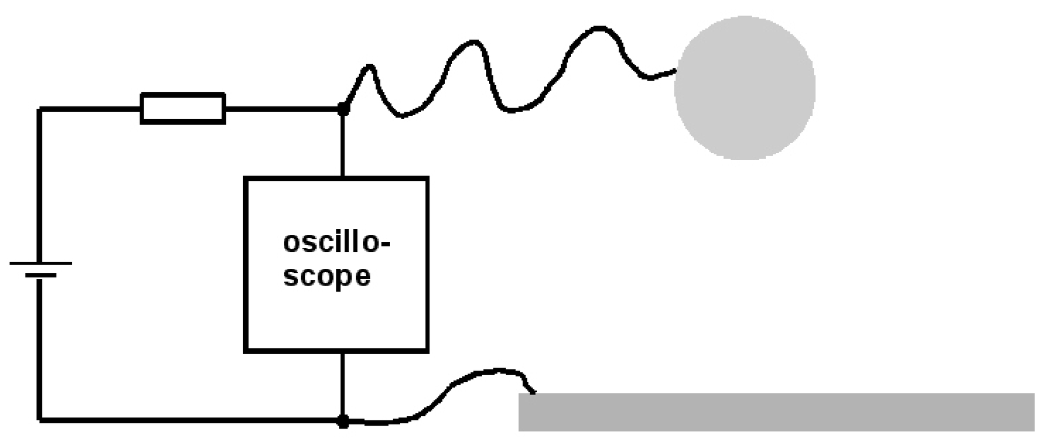


The used robot for performing large scale experiments.



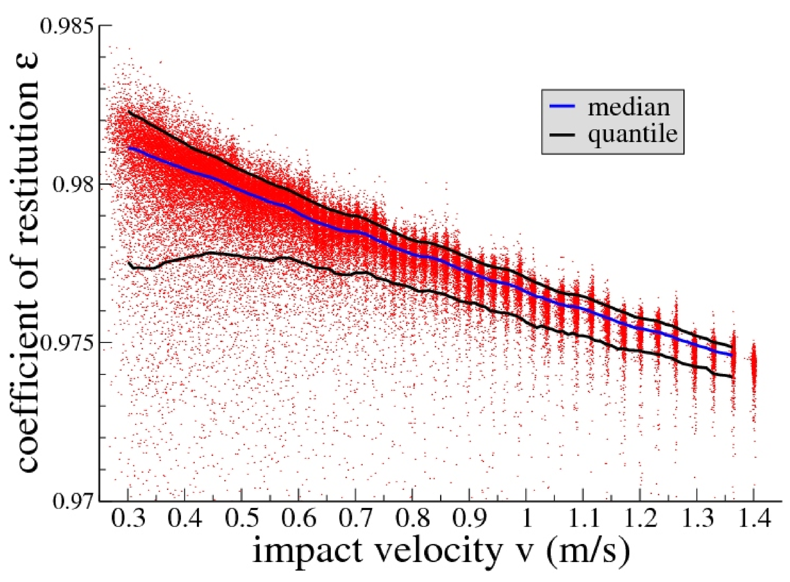


The robot records the sound caused by a ball bearing sphere hitting the glass plate. Three impacts are needed to calculate the coefficient of normal restitution $\varepsilon(v_1)=t_2/t_1$



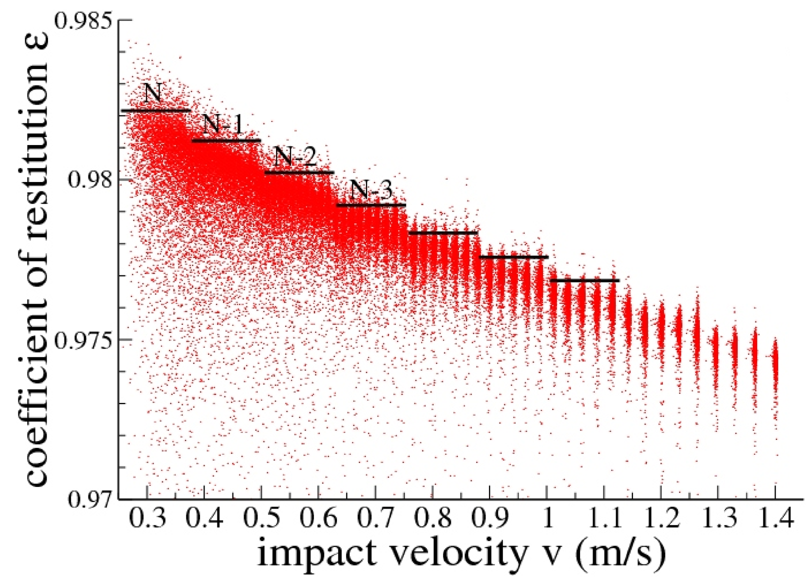
To measure the contact time a power source is connected with a silver coated glass plate and with a steel sphere.

Experimental data

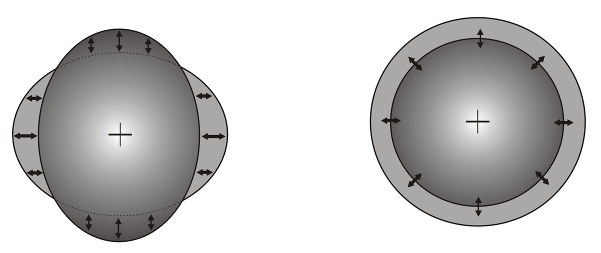


The plot on the left shows the coefficient of restitution of the steel sphere. As predicted theoretically the coefficient decays with increasing impact velocity. The distribution of ε is asymmetric with respects to larger and smaller values which can be seen by the distance of the upper and lower quantile (see Poster: Coefficient of normal restitution as a fluctuating quantity).

Performing a large number of independent measurements (here about 350000 data points) the data reveals a step-like structure. The steps are almost equidistant in length.

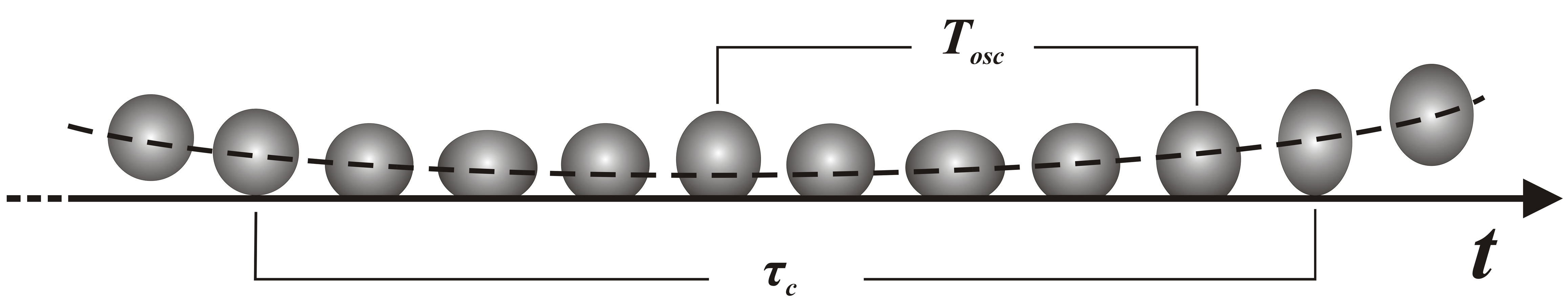


Discussion

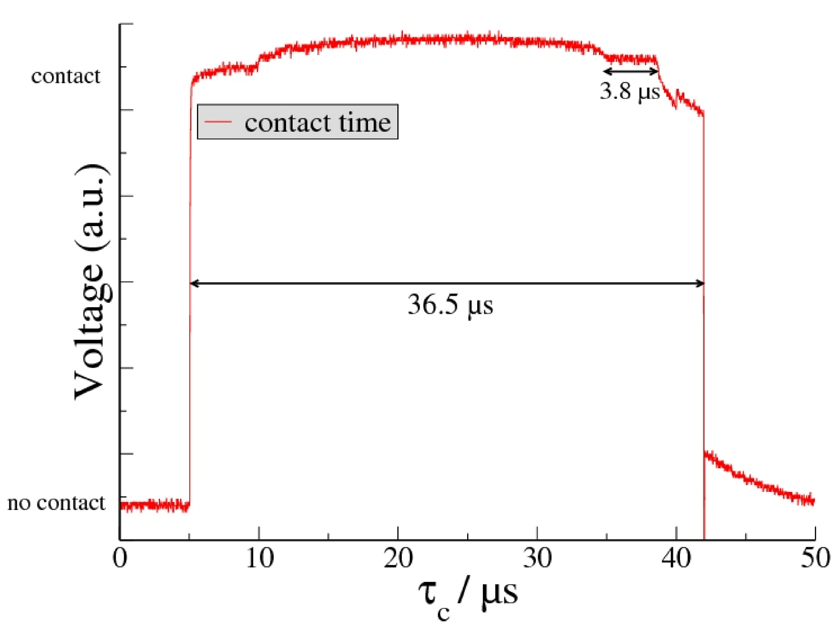


Sketch of the two main eigen modes of an oscillating sphere. For a 5 mm steel sphere as used in the experiment the corresponding eigen frequencies [2] are:

- prolate/oblate mode: $T_{prol,obl}$ = 3.1 μs (left)
- breathing mode: T_{breath} = 1.8 μs (right)



Sketch of an impacting sphere. The dashed line marks the centre of mass. As the time of contact τ_c is about 30 μs and the frequency of the oscillation T_{osc} is about 1 MHz, the area of contact varies during the impact. Even the loss of contact is possible, mainly close to the end or beginning of the impact. In dependence on the phase of oscillation, this can cause both, a shorter or a longer time of contact to be measured.



Measurement of the contact time. The contact duration is about 37 μs and in the same region as the theoretical value [3]:

$$\tau_c = \frac{2\tau^0_{\max} m_{\rm eff}^{2/5}}{\rho^{2/5} v^{1/5}}\,, \qquad (1)$$

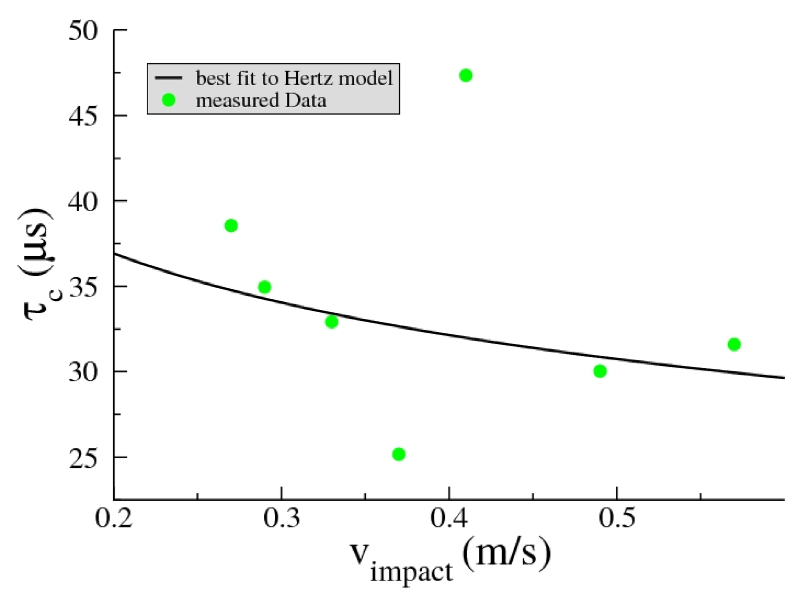
where ρ is a material parameter, $\tau^0_{max} \approx 1.609$ and m_{eff} is the effective mass.

The step-like structure of the curve is a fingerprint of the varying contact area. The length of the steps ($\approx$3.8 µs) is in good agreement with the theoretical prediction for the eigen frequencies.

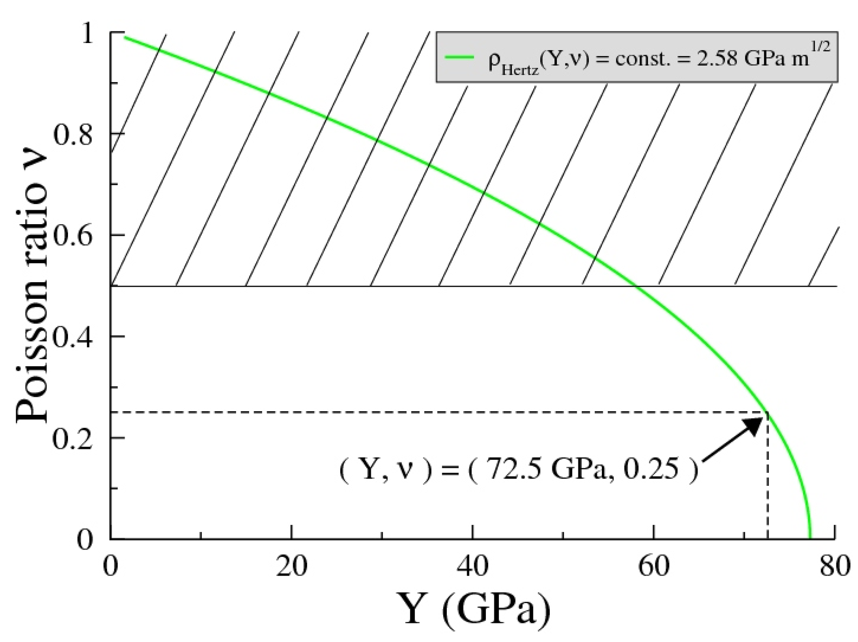
The right plot shows the experimentally measured contact times together with the theoretical value, Eq. (1), which follows from the Hertz model [4]. The material parameter ρ was used as a fit variable.

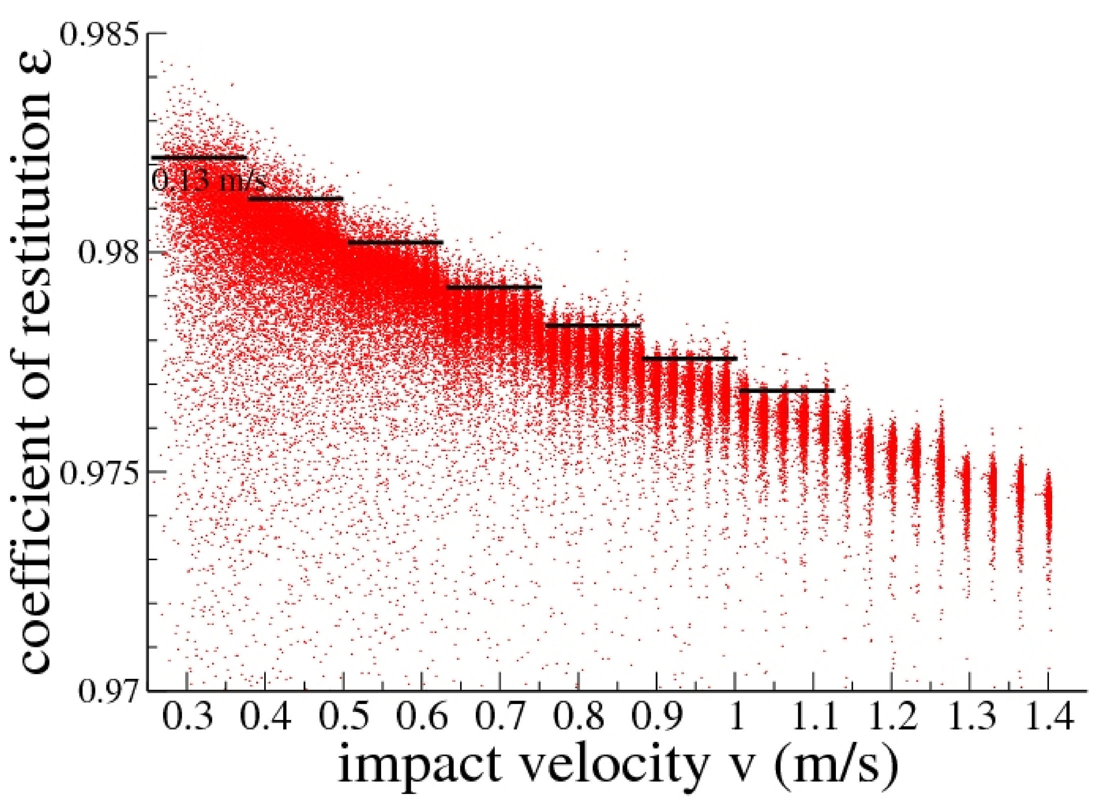
$$\rho = \frac{2Y R_{\rm eff}}{3(1-\nu^2)}\,, \qquad (2)$$

where Y is the Young modulus, R_{eff} the effective radius and ν is the Poisson ratio.



The best fit was found for ρ^{fit} = 2.58 GPa m$^{1/2}$. The plot (right) shows the combinations of Y and ν which yield ρ^{fit}. A Poisson ratio above 0.5 is not possible for a stable material. The theoretical prediction for the eigen frequencies were obtained for ν =1/4 (Cauchy assumption). Therefore we chose the combination (Y,ν) = (72.5 GPa,0.25). Y=72.5 GPa is in agreement with the value for steel. Thus, the measured contact times supports the hypothesis that the step-like structure of $\varepsilon=\varepsilon(v_{imp})$ is caused by the oscillation of the sphere.





The duration of contact τ_c of an ideal (non-oscillating) sphere with a flat plate depends on the impact rate v_{imp}. At such v_{imp} where the quantity int(τ_c/T_{osc}) changes the plot $\varepsilon(v_{imp})$ shows a step.
The length of the step (0.13 m/s) compares well with the theoretical prediction (0.138 m/s) following from the eigen frequencies of the oscillating sphere.

References:

[1] H. Maecker, Über die Bewegung gestoßener Körper, *Die Naturwissenschaften*, Erich v. Holst and Ernst Lamla editors, Springer Verlag, p. 521, (1956)
[2] A.E.H. Love, A treatise on the mathematical theory of elasticity, Dover Publications, p. 278, (1995)
[3] Thomas Schwager and Thorsten Pöschel, Coefficient of restitution for viscoelastic spheres: The effect of delayed recovery, *Phys. Rev. E.*, Preprint, (2008)
[4] H. Hertz, Über die Berührung fester elastischer Körper, *J. Reine Angewandte Math*, 94,156/71 (1882)