

Flux Saturation Length of Sediment Transport

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(Received 14 December 2012; published 20 November 2013)

Sediment transport along the surface drives geophysical phenomena as diverse as wind erosion and dune formation. The main length scale controlling the dynamics of sediment erosion and deposition is the saturation length L_s , which characterizes the flux response to a change in transport conditions. Here we derive, for the first time, an expression predicting L_s as a function of the average sediment velocity under different physical environments. Our expression accounts for both the characteristics of sediment entrainment and the saturation of particle and fluid velocities, and has only two physical parameters which can be estimated directly from independent experiments. We show that our expression is consistent with measurements of L_s in both aeolian and subaqueous transport regimes over at least 5 orders of magnitude in the ratio of fluid and particle density, including on Mars.

DOI: [10.1103/PhysRevLett.111.218002](https://doi.org/10.1103/PhysRevLett.111.218002)

PACS numbers: 45.70.-n, 47.55.Kf, 92.40.Gc

Sediment transport along the surface drives a wide variety of geophysical phenomena, including wind erosion, dust aerosol emission, and the formation of dunes and ripples on ocean floors, river beds, and planetary surfaces [1–6]. The primary transport modes are *saltation*, which consists of particles jumping downstream close to the ground at nearly ballistic trajectories, and *creep* (grains rolling and sliding along the surface). A critical parameter in sediment transport is the distance needed for the particle flux to adapt to a change in flow conditions, which is characterized by the *saturation length*, L_s . Predicting L_s under given transport conditions remains a long-standing open problem [6–10].

Indeed, L_s partially determines the dynamics of dunes, for instance, by dictating the wavelength of the smallest (“elementary”) dunes on a sediment surface [11,12] and the minimal size of crescent-shaped barchans [11,13]. Moreover, although flux saturation plays a significant role for the evolution of fluvial sediment landscapes [14], morphodynamic models used in hydraulic engineering usually treat L_s as an adjustable parameter [15]. The availability of an accurate theoretical expression predicting L_s for given transport conditions would thus be an important contribution to the planetary, geological, and engineering sciences. In this Letter, we present such a theoretical expression for L_s . In contrast to previously proposed relations for L_s , the expression presented here explicitly accounts for the relevant forces that control the relaxation of particle and fluid velocities, and also incorporates the distinct entrainment mechanisms prevailing in aeolian and subaqueous transport (defined below).

The average momentum of transported grains per unit soil area, the sediment flux Q , is defined as $Q = MV$,

where M is the mass of sediment in flow per unit soil area, and V is the average particle velocity. Since the fluid loses momentum to accelerate the particles, Q is limited by a steady-state value, the saturated flux Q_s . This flux is largely set by the fluid density ρ_f and the fluid shear velocity u_* [1–4,6], which is proportional to the mean flow velocity gradient in turbulent boundary layer flow [6]. In typical situations, such as on the streamward side of dunes, the deviation of Q from Q_s is small, that is, $|1 - Q/Q_s| \ll 1$ [10,11,16]. The rate $\Gamma(Q)$ of the relaxation of Q towards Q_s in the downstream direction (x) can thus be approximately written as [7,10,16],

$$\Gamma(Q) = dQ/dx \approx [Q_s - Q]/L_s, \quad (1)$$

where Γ is Taylor expanded to first order around $Q = Q_s$ [$\Gamma(Q_s) = 0$], and the negative inverse Taylor coefficient gives the saturation length, L_s . Flux saturation is controlled by the downstream evolutions of M and V towards their respective steady-state values, M_s and V_s . Changes in M with x are controlled by particle entrainment from the sediment bed into the transport layer. In the aeolian regime (dilute fluid such as air), entrainment occurs predominantly through particle impacts [6], whereas in the subaqueous regime (dense fluid such as water) entrainment occurs mainly through fluid lifting [2,3]. On the other hand, the evolution of V towards V_s is mainly controlled by the acceleration of the particles due to fluid drag, and their deceleration due to grain-bed collisions [10,12]. We note that the evolution of V is affected by changes in M and vice versa. For instance, an increase of M leads to a decrease in V in the absence of horizontal forces due to conservation of horizontal momentum. For simplicity, previous studies

neglected either the saturation of V [10,17] or the relaxation of M , as well as changes in V due to grain-bed collisions [7,12]. Moreover, all previous studies did not account for the relaxation of the fluid velocity (U) towards its steady-state value (U_s) within the transport layer. This relaxation is driven by changes in the transport-flow feedback resulting from the relaxations of M and V . For instance, increasing V reduces the relative velocity $V_r = U - V$ and thus the fluid drag. In turn, as V_r decreases, the amount of momentum transferred from the fluid to the transport layer also decreases, which results in an increase in U , whereas an increase in U again increases V_r .

In this Letter, we derive a theoretical expression for L_s which encodes *all* aforementioned relaxation mechanisms. Indeed, since previously proposed relations for L_s neglect some of the interactions that determine L_s [7,10–12], it is uncertain how to adapt these equations to compute L_s in extraterrestrial environments, such as Mars [5,6,13]. Our theoretical expression overcomes this problem, since it is valid for arbitrary physical environments for which turbulent fluctuations of the fluid velocity, and thus transport as suspended load [6], can be neglected. For aeolian transport under terrestrial conditions, this regime corresponds to $u_* \leq 4u_t$, where u_t is the threshold u_* for sustained transport [2,3,6].

We start from the momentum conservation equation for steady ($\partial/\partial t = 0$) dilute granular flows [18],

$$\partial \rho \langle v_x^2 \rangle / \partial x + \partial \rho \langle v_x v_z \rangle / \partial z = \langle f_x \rangle, \quad (2)$$

where $\langle \rangle$ denotes the ensemble average, ρ the mass density, $\mathbf{v}$ the particle velocity, and $\mathbf{f}$ the external body force per unit volume applied on a sediment particle. Here, $\mathbf{f}$ incorporates the main external forces acting on the transported particles: drag, gravity, buoyancy, and added mass. The added mass force arises because the speed of the fluid layer immediately surrounding the particle is closely coupled to that of the particle, thereby enhancing the particle's inertia by a factor $1 + 0.5s^{-1}$, where $s = \rho_p/\rho_f$ is the grain-fluid density ratio [2]. Although this added mass effect is negligible in aeolian transport ($0.5s^{-1} \ll 1$), it affects the motion of particles in the subaqueous regime [2]. Integration of Eq. (2) over the entire transport layer depth ($\int_0^\infty \dots dz$) yields,

$$\frac{d(c_v MV^2)}{dx} = \int_0^\infty \langle f_x \rangle dz + (\rho \langle v_x v_z \rangle)(0), \quad (3)$$

where $M = \int_0^\infty \rho dz$, $V = \int_0^\infty \rho \langle v_x \rangle dz / M$, and $c_v = \int_0^\infty \rho \langle v_x^2 \rangle dz / (MV^2)$. In Eq. (3), the quantity $(\rho \langle v_x v_z \rangle)(0)$ gives the difference between the average horizontal momentum of particles impacting onto $[-(\rho \langle v_x v_z \rangle)(0)]$ and leaving $[(\rho \langle v_x v_z \rangle)(0)]$ the sediment bed per unit time and soil area. This momentum change is consequence of the collisions between particles within the sediment bed ($z \leq 0$). Thus, $(\rho \langle v_x v_z \rangle)(0)$ is an effective frictional force which the soil applies on the transport layer per unit soil

area. It is proportional to the normal component of the force which the transport layer exerts onto the sediment bed [3,10,19], $(\rho \langle v_x v_z \rangle)(0) = -\mu g M(s-1)/(s+0.5)$, where μ is the associated Coulomb friction coefficient, and g the gravitational constant. In order to obtain the momentum conservation equation of the particles within the transport layer from Eq. (3), we first note that $\int_0^\infty \langle f_x^{\text{drag}} \rangle dz \approx (3M/4sd)C_d(V_r)V_r^2$ [19], where d is the mean grain diameter, while $C_d(V_r)$ is the drag coefficient associated with the fluid drag on transported particles, which is intermediate to fully viscous drag ($C_d \propto \nu/[V_r d]$, with ν standing for the kinematic viscosity) and fully turbulent drag (constant C_d). By further noting that the change of c_v with x is negligible (see Supplemental Material [20]), we obtain,

$$c_v \frac{d(MV^2)}{dx} = \frac{3M}{4(s+0.5)d} C_d(V_r)V_r^2 - \frac{s-1}{s+0.5} \mu g M. \quad (4)$$

Next, we solve Eq. (4) for dV/dx thus obtaining an equation of the form $(dV/dx) = \Omega(V)$, and we expand $\Omega(V)$ around saturation, that is, $\Omega(V) \approx \Omega(V_s) + (V - V_s)d\Omega/dV|_{V_s}$. By noting that $\Gamma(V) = (dQ/dx)(V) = (M(V) + V(dM(V)/dV))\Omega(V)$ and $\Omega(V_s) = 0$, we obtain $L_s = -(d\Gamma/dQ)^{-1}|_{Q=Q_s} = -(d\Omega/dV)^{-1}|_{V=V_s}$, which leads to,

$$L_s = (s+0.5)c_v(2+c_M)V_s V_{rs} FK[\mu(s-1)g]^{-1}, \quad (5)$$

where $c_M = (V_s/M_s)(dM/dV)(V_s)$, and $K = (1 - (dU/dV)(V_s))^{-1}$, while V_{rs} (the steady-state value of V_r) and F are given by,

$$V_{rs} = [\sqrt{8\mu(s-1)gd/9 + (8\nu/d)^2} - 8\nu/d], \quad \text{and}, \quad (6)$$

$$F = [V_{rs} + 16\nu/d][2V_{rs} + 16\nu/d]^{-1}, \quad (7)$$

respectively. Equations (6) and (7) result from using $C_d(V_r) = (24\nu/V_r d) + 1.5$ (valid for natural sediment [21]). We find that using other reported drag laws only marginally affects the value of L_s . Furthermore, we note that in the subaqueous regime $c_M \approx 0$, since in this regime M changes within a time scale which is more than 1 order of magnitude larger than the time scale over which Q changes [22]. This difference in time scales implies $VdM \ll dQ$ and thus $VdM \ll MdV$ in the subaqueous regime. In contrast, in the aeolian regime, $c_M \approx 1$ as the total mass of ejected grains upon grain-bed collisions is approximately proportional to the speed of impacting grains [23], which yields $M/M_s \approx V/V_s$.

In Eq. (5), the quantity K encodes the effect of the relaxation of the transport-flow feedback, neglected in previous works [7,10,17]. In the subaqueous regime, this transport-flow feedback has a negligible influence on the fluid speed [22] (and thus on its relaxation). In this regime, $(dU/dV)(V_s) \approx 0$, which yields $K \approx 1$ and thus,

$$L_s^{\text{subaq}} = [2s + 1]c_v V_s V_{rs} F[\mu(s - 1)g]^{-1}. \quad (8)$$

In contrast, in the aeolian regime, U scales with the shear velocity at the bed (u_b) [19,22], and thus $(dU/dV)(V_s) \approx (U_s/u_{bs})(du_b/dV)(V_s)$, where u_{bs} is the steady-state value of u_b . Using the mixing length approximation of inner turbulent boundary layer equations [24], u_b can be expressed as $u_b = u_*[1 - 3MC_d(V_r)V_r^2/(4(s + 0.5)d\rho_f u_*^2)]^{1/2}$ [22]. By using this expression to compute du_b/dV and noting that $u_{bs} \approx u_t$ [6], we obtain the following expression for K ,

$$K = \frac{1 + F^{-1}[(V_s + V_{rs})/(2V_{rs})][(u_*/u_t)^2 - 1]}{1 + [(V_s + V_{rs})/(2V_s)][(u_*/u_t)^2 - 1]}. \quad (9)$$

Using Eq. (9) to compute K , L_s in the aeolian regime of transport $[(s + 0.5)/(s - 1) \approx 1]$ is then given by

$$L_s^{\text{aeolian}} = 3c_v V_s V_{rs} F K [\mu g]^{-1}. \quad (10)$$

We show in Section IV of the Supplemental Material [20] that Eq. (10) can be approximated by the simpler form of $L_s^{\text{aeolian}} \approx 3c_v V_s^2 [\mu g]^{-1}$ in the limit of large u_*/u_t .

Therefore, from our general expression for L_s [Eq. (5)] we obtain two expressions—Eqs. (8) and (10)—which can be used to predict L_s in the subaqueous and aeolian transport regimes, respectively. Both use only two parameters, namely μ and c_v , which are estimated from independent measurements. Specifically, μ is estimated from measurements of M_s and Q_s for different values of u_* in air and under water, while c_v is estimated from measurements of the particle velocity distribution [20,25,26]. From these experimental data, we obtain $\mu \approx 1.0$ (0.5) and $c_v \approx 1.3$ (1.7) for the aeolian (subaqueous) regime.

Both Eqs. (8) and (10) are consistent with the behavior of L_s with u_* observed in experiments. Indeed, L_s mainly depends on u_* via the average particle velocity, V_s . For subaqueous transport, in which V_s is a linear function of u_* , L_s varies linearly with V_s and thus with u_* , which is consistent with experiments [8]. In contrast, V_s depends only weakly on u_* for aeolian transport [6,22]. Consequently, L_s is only weakly dependent on u_* in this regime, which is also consistent with experiments [7]. In fact, when neglecting this weak dependence on u_* , Eq. (10) reduces to $L_s \propto sd$ [7,12] in the limit of large particle Reynolds numbers $\sqrt{sgd^3}/\nu$ for which $V_s \propto \sqrt{sgd}$ [22]. Moreover, we estimate the average particle velocity V_s as a function of u_*/u_t using well-established theoretical expressions which were validated against experiments of sediment transport in the aeolian or in the subaqueous regime. Specifically, we use the model of Ref. [19] for obtaining $V_s(u_*/u_t)$ in the aeolian regime and the model of Ref. [25] for the subaqueous regime [20].

The squares in Fig. 1 denote wind tunnel measurements of L_s for different values of u_* . These data were obtained by fitting Eq. (1) to the downstream evolution of the sediment flux, $Q(x)$, close to equilibrium [7]. Further estimates of L_s for aeolian transport under terrestrial

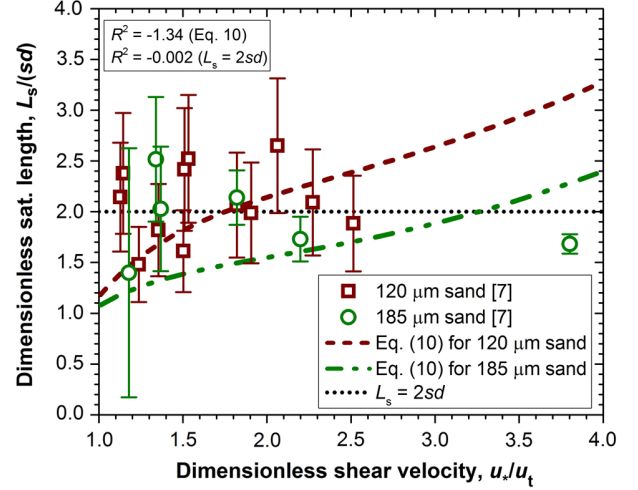


FIG. 1 (color online). Dimensionless saturation length, $L_s/(sd)$, versus u_*/u_t for aeolian transport under terrestrial conditions. Brown squares denote estimates of L_s from wind-tunnel measurements ($d = 120 \mu\text{m}$), while the error bars are due to uncertainties in the measurements of the sediment flux [7]. Green circles denote L_s obtained from the wavelength of elementary dunes on top of large barchans ($d = 185 \mu\text{m}$), whereas the error bars contain uncertainties in the dune size [7] (potential systematic uncertainties [20] are not included). The colored lines represent predicted values of L_s using Eq. (5) for the corresponding experimental conditions ($\rho_p = 2650 \text{ kg/m}^3$, $\rho_f = 1.174 \text{ kg/m}^3$ and $\nu = 1.59 \times 10^{-5} \text{ m}^2/\text{s}$). The dotted horizontal line indicates the prediction of L_s using $L_s = 2sd$ [7,12]. The upper legend displays the corresponding values of the coefficient of determination, $R^2 = 1 - (\sum_i (L_{si}^{\text{measured}} - L_{si}^{\text{predicted}})^2 / \sum_i (L_{si}^{\text{measured}} - L_s^{\text{mean}})^2)$, which is a measure of a theory's ability to capture variation in data, with $R^2 = 1$ corresponding to a perfect fit).

conditions have been obtained from the wavelength (λ) of elementary dunes on top of large barchans [7,20]. These estimates correspond to the circles in Fig. 1, whereas the coloured lines in this figure denote $L_s/(sd)$ versus u_*/u_t predicted by Eq. (10). As we can see in Fig. 1, in spite of the scatter in the data, Eq. (10) yields reasonable agreement with the experimental data without requiring any fitting to these data. In contrast, the scaling $L_s = 2sd$ [7,12] was obtained from a fit to the data displayed in Fig. 1. Moreover, Fig. 2 shows values of L_s estimated from experiments on subaqueous transport under different shear velocities (symbols). These estimates were obtained from measurements of λ [12,27] and from the minimal cross-stream width, $W \approx 12L_s$ [13], of barchans in a water flume [8]. The colored lines show the behavior of L_s with u_* as predicted from Eq. (8) for subaqueous sand transport. We note that Eq. (8) is the first expression for L_s that shows good agreement with measurements of L_s under water. Indeed, the scaling relation $L_s = 2sd$ does not capture the increasing trend of L_s with u_*/u_t evident from the experimental data.

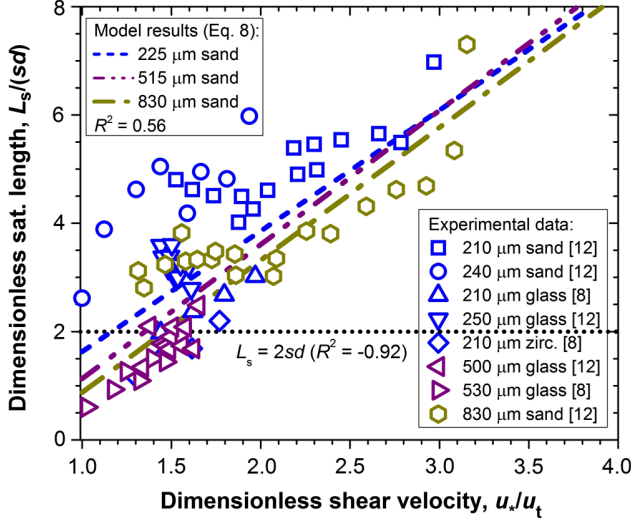


FIG. 2 (color online). $L_s/(sd)$ versus u_*/u_t for subaqueous transport. Symbols denote estimates of L_s from the wavelength of elementary dunes [12] and from the minimal cross-stream width of subaqueous barchans, $W \approx 12L_s$ [8]. The colored lines denote predicted values of L_s using Eq. (5) for subaqueous transport of sand ($\rho_p = 2650 \text{ kg/m}^3$, $\rho_f = 10^3 \text{ kg/m}^3$ and $\nu = 10^{-6} \text{ m}^2/\text{s}$), with grain sizes roughly matching those used in the experiments. The dotted horizontal line indicates the prediction of L_s using the scaling $L_s = 2sd$ [7,12]. The values of R^2 (coefficient of determination) for both expressions are also shown.

An excellent laboratory for further testing our model is the surface of Mars, where the ratio of grain to fluid density (s) is about 2 orders of magnitude larger than on Earth. We estimate the Martian L_s from reported values of the minimal crosswind width W of barchans at the Arkhangelsky crater in the southern highlands and at a dune field near the north pole [13,20]. However, using Eq. (10) to predict L_s on Mars is difficult because both the grain size d and the typical shear velocity u_{*typ} for which the dunes were formed are poorly known. Indeed, we need to know both quantities to calculate V_s [19]. We thus predict the Martian L_s using a range of plausible values of d and u_{*typ} . Specifically, we assume d to lie in the broad range of 100–600 μm based on recent studies [5]. Estimating u_{*typ} on Mars is also difficult, both because of the scarcity of wind speed measurements [28], and because the threshold u_* required to initiate transport (u_{ft}) likely exceeds u_t by up to a factor of ~ 10 [6,19,29]. We therefore calculate L_s for two separate estimates of u_{*typ} : the first using $u_{*typ} = u_{ft}$, consistent with previous studies [13,30], and the second calculating u_{*typ} based on the wind speed probability distribution measured at the Viking 2 landing site [20], which results in an estimate of u_{*typ} closer to u_t . Figure 3 shows that the values of L_s predicted with either of these estimates are consistent with those estimated from the minimal barchan width. This good agreement suggests

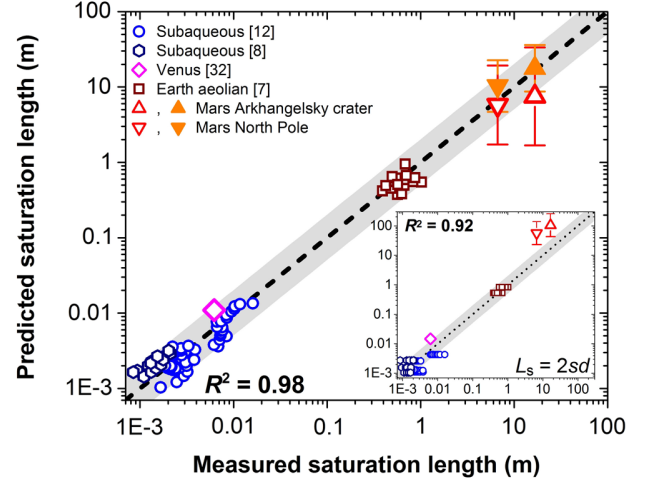


FIG. 3 (color online). Comparison of measured and predicted values of L_s for various environments; the gray shading denotes agreement between measurements and Eqs. (10) and (8) within a factor of two. Aeolian and subaqueous data were obtained as described in Figs. 1 and 2. For Venus, L_s was estimated from the wavelength of elementary dunes [20]. For Mars, L_s was derived from estimates of the minimal size of barchans in two Martian dune fields [20], and plotted against the predicted L_s for a range of plausible dune particle sizes ($d = 100\text{--}600 \mu\text{m}$ [5]) and for two separate estimates of u_{*typ} (see text). The error bars denote the range in predicted L_s arising from the range in d [5], and the symbols denote the geometric mean. The filled orange symbols use $u_{*typ} = u_{ft}$ [13,30], while the open red symbols use the u_{*typ} calculated from Viking 2 wind speed measurements [20]. The inset shows measured and predicted values of L_s for the same conditions as in the main plot, but using $L_s = 2sd$ [7,12]. Values of R^2 (coefficient of determination) in each plot were calculated in log10-space such that each data point was weighted equally.

that the previously noted overestimation of the minimal size of Martian dunes [31] is largely resolved by accounting for the low Martian value of u_t/u_{ft} [19] and the proportionally lower value of the particle speed V_s , as hypothesized in Ref. [29]. Indeed, the scaling $L_s = 2sd$ (inset of Fig. 3) requires $d \approx 29 \mu\text{m}$ and $d \approx 40 \mu\text{m}$ to be consistent with L_s for the north polar and Arkhangelsky dune fields, respectively. However, such particles are most likely transported as a suspended load on Mars [30], as they are on Earth [4].

Finally, Fig. 3 also compares Eq. (8) to measurements of L_s for Venusian transport, which have been estimated from the wavelength of elementary dunes produced in a wind-tunnel mimicking the Venusian atmosphere [32].

In conclusion, Eq. (5) is the first expression capable of quantitatively reproducing measurements of the saturation length L_s under different flow conditions in both air and under water, and is in agreement with measurements over at least 5 orders of magnitude of variation in the sediment to fluid density ratio. The future application of this expression thus has the potential to provide important contributions to

calculate sediment transport, the response of saltation-driven wind erosion and dust aerosol emission to turbulent wind fluctuations, and the dynamics of sediment-composed landscapes under water, on Earth's surface and on other planetary bodies. The code to calculate L_s with our model is available from the first author.

We acknowledge support from Grants No. NSFC 41350110226, No. NSFC 41376095, No. ETH-10-09-2, No. NSF AGS 1137716, from the European Research Council (ERC) Advanced Grant No. 319968-FlowCCS, and DFG through the Cluster of Excellence "Engineering of Advanced Materials." We thank Miller Mendoza and Robert Sullivan for discussions, and Jeffery Hollingsworth for providing us with the pressure and temperature at the Martian dune fields.

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— Supplemental Material —

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This Supplemental Material is organized as follows. In Section I we derive the values of the steady-state particle speed square correlation c_v and the Coulomb friction coefficient μ — the only parameters of our theoretical expression of the saturation length — from independent experiments of sediment transport in air and under water. Moreover, in Section II we show the analytical expressions used to calculate the steady-state average particle velocity V_s in the aeolian regime and in the subaqueous regime of transport. Next, in Section III, we discuss our estimation of the saturation length from the scale of dunes in different environments. We then present the values of L_s estimated from the size of dunes on Earth, on Mars, on Venus and under water, which we use to compare with the predictions from our theoretical expression for L_s in Fig. 3 of the paper. Finally, in Section IV, we compare the predictions for L_s on Earth and Mars obtained from our theoretical expression for L_s in the aeolian regime (Eq. (10) of the paper) with the predictions obtained using a simplified version of Eq. (10), which is valid for large values of u_*/u_t .

I. ESTIMATING c_v AND μ FROM EXPERIMENTAL DATA

Our theoretical expression for L_s has two empirical quantities which must be estimated from experimental measurements. These empirical quantities are the Coulomb friction coefficient (μ) and the particle speed square correlation (c_v). In this Section, we describe how we estimate these quantities from existing experimental data of sediment flux in both aeolian and subaqueous transport regimes.

A. The particle speed square correlation, c_v

The quantity c_v , that is, the particle speed-square correlation, is defined by the equation,

$$c_v = \frac{\int_0^\infty \rho \langle v_x^2 \rangle dz}{MV^2} = \frac{M \int_0^\infty \rho \langle v_x^2 \rangle dz}{\left(\int_0^\infty \rho \langle v_x \rangle dz \right)^2}, \quad (1)$$

where $M = \int_0^\infty \rho dz$, $V = \int_0^\infty \rho \langle v_x \rangle dz / M$, ρ is the particle mass density, $\mathbf{v}$ is the particle velocity, and $\langle \rangle$ denotes ensemble averaging. That is, the normalized variance of the velocity distribution of the transported particles is then written as $\delta_v = c_v - 1$, where c_v is given by Eq. (1).

First, we note that for transport in the equilibrium ($\partial/\partial x = 0$), $\left(\int_{-\infty}^\infty \rho \langle v_x \rangle dz \right)^2 / \int_{-\infty}^\infty \rho \langle v_x^2 \rangle dz$ is proportional to $u_*^2 - u_t^2$ for both transport regimes, where u_t is the transport threshold [1]. Hence, since most of the transport occurs above the sediment bed ($z > 0$), also the quantity $\left(\int_0^\infty \rho \langle v_x^2 \rangle dz \right)^2 / \int_0^\infty \rho \langle v_x \rangle dz$ is nearly proportional to $u_*^2 - u_t^2$ in the equilibrium. On the other hand, it is known from experiments that M_s approximately scales with $u_*^2 - u_t^2$ [2–4]. Therefore, due to Eq. (1), c_v is nearly independent of u_* for equilibrium transport. By considering that c_v is nearly independent of u_* for equilibrium transport, as discussed above, it seems reasonable that changes of c_v with x during the saturation process of the sediment flux close to the equilibrium can be regarded as negligible compared to the corresponding changes of M or V with x . In this manner, we disregard changes of c_v with x in the derivation of the saturation length equation, as mentioned in the main document (cf. text before Eq. (4)). In other words, the value of c_v used in the saturation length equation corresponds to the *steady-state* value of the particle speed square correlation. In the following we show how we estimate c_v for transport in the aeolian and subaqueous regimes. For this purpose, it is helpful to rewrite c_v as,

$$c_v = \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2}, \quad (2)$$

where the overbar denotes the height average of a quantity according to the expression,

$$\overline{A} = \frac{\int_0^\infty A \rho dz}{\int_0^\infty \rho dz} = \frac{\int_0^\infty A \rho dz}{M}. \quad (3)$$

1. Aeolian regime

It was shown from experiments of sediment transport

in a wind tunnel [3] that the particle concentration profile $\rho(z)$ and the particle velocity profile $\langle v_x \rangle(z)$ behave according to the following equations,

$$\rho(z) = \rho(0)e^{-z/z_\rho}, \quad (4)$$

$$\langle v_x \rangle(z) = \langle v_x \rangle(0) + mz, \quad (5)$$

where $z_\rho \approx 10$ mm, $\langle v_x \rangle(0) \approx 1$ m/s, and $m \approx 70$ s⁻¹ are roughly constant with u_* . Inserting Eqs. (4) and (5) in Eq. (1), we obtain,

$$c_v = \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2} = \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2} \times \frac{\overline{\langle v_x \rangle^2}}{\overline{\langle v_x \rangle}^2} = \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2} \times \frac{\int_0^\infty \rho dz \int_0^\infty \rho \langle v_x \rangle^2 dz}{\left(\int_0^\infty \rho \langle v_x \rangle dz \right)^2} = \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2} \left(1 + \left(\frac{mz_\rho}{\langle v_x \rangle(0) + mz_\rho} \right)^2 \right) \approx 1.17 \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2}. \quad (6)$$

The value of $\overline{\langle v_x^2 \rangle} / \overline{\langle v_x \rangle}^2$ can be estimated from the experimental results of Ref. [5]. These authors reported a histogram of the horizontal velocity v_x of the particles located at a height $z_h = 2$ cm (see Fig. 13 of Ref. [5]). From the results of their experiments, we obtain,

$$\frac{\langle v_x^2 \rangle(z_h)}{\langle v_x \rangle^2(z_h)} \approx 1.1. \quad (7)$$

Furthermore, it was shown that, in the aeolian regime of transport, the normalized distribution of the horizontal velocity of the particles within the transport layer does not vary much with the height [6]. Therefore, based on this experimental observation, we use the result of Eq. (7) to compute $\overline{\langle v_x^2 \rangle} / \overline{\langle v_x \rangle}^2$ for all values of z within the transport layer. In doing so, we obtain the following estimate for c_v in the aeolian regime of transport,

$$c_v^{\text{aeolian}} \approx 1.17 \frac{\overline{\langle v_x^2 \rangle}}{\overline{\langle v_x \rangle}^2} \approx 1.17 \frac{\langle v_x^2 \rangle(z_h)}{\langle v_x \rangle^2(z_h)} \approx 1.3. \quad (8)$$

2. Subaqueous regime

We estimate c_v for transport in the subaqueous regime from measurements of the distribution $P_v(v_x)$ of the horizontal velocities v_x of particles in sediment transport under water [4]. These measurements were conducted using particles of average diameter $d = 2.24$ mm and under rescaled shear velocity $u_*/u_t = 2.1$ [4]. In order to compute $P_v(v_x)$ from the particle trajectories, the particles were considered as being transported if they had a velocity larger than a certain cut-off value, v_c [4]. The distribution of horizontal velocities for the transported particles was fitted using an exponential distribution,

$$P_v(v_x) = \frac{1}{V_f} \exp \left[-\frac{v_x - v_c}{V_f} \right], \quad (9)$$

where $V_f \approx 110$ mm/s. By using this distribution, we can compute c_v as,

$$c_v = \frac{\int_{v_c}^\infty v_x^2 P_v(v_x) dv_x}{\left(\int_{v_c}^\infty v_x P_v(v_x) dv_x \right)^2} = \frac{1 + \left(1 + \frac{v_c}{V_f} \right)^2}{\left(1 + \frac{v_c}{V_f} \right)^2}. \quad (10)$$

Ref. [4] did not report specific values of v_c corresponding to specific measurements of u_*/u_t [4]. Instead the authors mentioned that v_c is within the range between 10 mm/s and 30 mm/s, depending on the water flow rate. In Ref. [4], $P_v(v_x)$ was obtained using $d = 2.24$ mm and $u_*/u_t = 2.1$, which correspond to intermediate values for d and u_*/u_t investigated in the experiments [4]. Therefore, in order to compute c_v using the horizontal velocity distribution $P_v(v_x)$ obtained in these experiments, we use the intermediate value $v_c = 20$ mm/s as an approximate estimate for the average cut-off velocity. Using this estimate for v_c , Eq.(10) yields,

$$c_v^{\text{subaqueous}} \approx 1.7, \quad (11)$$

which is the value of steady-state particle speed square correlation for transport in the subaqueous regime.

B. The Coulomb friction coefficient, μ

As mentioned in the main document, the Coulomb friction coefficient μ is defined by the relation,

$$(\rho \langle v_x v_z \rangle)(0) = -\mu(s-1)gM/(s+0.5). \quad (12)$$

The left-hand-side of Eq. (12) is the grain shear stress at the bed which is associated with dilute granular flows, $\tau_g = -(\rho \langle v_x v_z \rangle)(0)$ [7]. That is,

$$\tau_g = \tau - \tau_{fs}, \quad (13)$$

where τ_{fs} is the fluid shear stress at the bed. Indeed, different studies showed that, for equilibrium transport ($M = M_s$), τ_g can be expressed as,

$$\tau_g \approx \tau - \tau_t, \quad (14)$$

whereas this approximation has been verified both for transport in the aeolian regime [2, 8] and for transport in the subaqueous regime [1]. By inserting Eq. (13) into Eq. (12), the following expression for M_s is obtained,

$$M_s = \frac{s+0.5}{\mu(s-1)g} \cdot [\tau - \tau_{fs}]. \quad (15)$$

This equation can be then rewritten, using the approximation in Eq. (14), as,

$$M_s \approx \frac{s+0.5}{\mu(s-1)g} \cdot [\tau - \tau_t]. \quad (16)$$

Indeed, from experiments on sediment transport in the subaqueous regime [4] it was found, using video-imaging techniques, that M_s behaves according to the expression,

$$M_s = \frac{s}{0.415(s-1)g} \cdot [\tau - \tau_t]. \quad (17)$$

Therefore, by comparing Eqs. (16) and (17) with $(s + 0.5)/s = 1.19$ valid for subaqueous sediment transport ($s = 2.65$), we obtain $\mu/1.19 = 0.415$. Thus, in the *subaqueous regime of transport*, the Coulomb friction coefficient has the approximate value,

$$\mu_{\text{subaqueous}} \approx 0.5. \quad (18)$$

Indeed, values within the range between $\mu/1.19 = 0.3$ and $\mu/1.19 = 0.5$ — and thus consistent with the value of μ estimated above — have been reported from measurements of particle trajectories in the subaqueous sediment transport [9–11].

Moreover, for the *aeolian regime of transport*, the value,

$$\mu_{\text{aeolian}} \approx 1.0, \quad (19)$$

was found in a previous work [2] through determining μ indirectly both from fitting a sediment transport model which includes Eq. (16) to experimental sediment flux data of Creyssels et al. [3]. We note that μ is the inverse of the parameter α in Ref. [2].

II. ANALYTICAL EXPRESSIONS FOR CALCULATING THE SATURATED AVERAGE PARTICLE VELOCITY V_s

In order to compute the saturation length of sediment transport, the average sediment velocity V_s must be known. We use well-established theoretical expressions reported in the literature to compute V_s in the aeolian regime or in the subaqueous regime.

Aeolian regime — For aeolian transport, we estimate V_s using the model of Ref. [2] since this model showed excellent agreement with measurements of Q_s performed in the experiments of Creyssels et al. [3] — from which we also determined the Coulomb friction coefficient μ (Section IB). Ref. [2] gave the following expression for V_s ,

$$V_s = V_t + [3u_t/2\kappa] \cdot \ln(V_s/V_t) + [u_*/\kappa] \cdot F_\gamma(u_t/u_*) \quad (20)$$

and $u_t = \kappa(V_{rs} + V_o) \cdot [(1 - \eta) \cdot \ln(z_{mt}/z_o)]^{-1}$, with,

$$F_\gamma(x) = (1 - x) \cdot \ln(1.78\gamma) + 0.5(1 - x^2) \cdot E_1(\gamma) + 1.154(1 + x \ln x)(1 - x)^{2.56},$$

$V_t = V_s(u_t) = V_o + \eta V_{rs}/[1 - \eta]$ and $z_{mt} = \beta\gamma V_{rs}^{\frac{1}{2}} V_t^{\frac{3}{2}} \cdot [\mu g]^{-1}$, while $V_o = 16.2\sqrt{g_f d}$, with $g_f = g + 6\zeta/[\pi\rho_p d^2]$ [2]. Furthermore, $E_1(x)$ is the exponential integral function, $\kappa = 0.4$ is the von Kármán constant, $\zeta = 5 \times 10^{-4}$ N/m encodes the influence of cohesion [2], $\beta =$

0.095, $\gamma = 0.17$, and $\eta = 0.1$ are empirically determined parameters [2], and the surface roughness of the quiescent sediment bed is given by $z_o = d \exp(-\kappa B)$, with $B = 8.5 + [2.5 \ln(R_p) - 3] \exp\{-0.11[\ln(R_p)]^{2.5}\}$ and $R_p = u_t d/\nu$ [2].

Subaqueous regime — It has been verified in a large number of experimental studies [4, 12–17] that the equilibrium particle velocity in the subaqueous regime of transport approximately follows the expression,

$$V_s = au_* - V_{rs}, \quad (21)$$

where a is a dimensionless number. We note that the above expression is consequence of the equation, $V_s = U_s - V_{rs}$, where U_s is taken proportional to u_* . Eq. (21) has been fitted to experimental data assuming $V_{rs} = au_t - b\sqrt{(s-1)gd}$, where b is another dimensionless number [4]. This assumption is valid for sufficiently large grain sizes d for which effects of the viscosity ν on V_{rs} can be neglected. However, the experimental estimates of the saturation length in the subaqueous regime were obtained from experiments using particles of diameters down to four times smaller than the particle size considered in Ref. [4]. Therefore, instead of using the approximation for V_{rs} given above, we take Eq. (6) of the paper to calculate V_{rs} . In Fig. 1 we show that using Eq. (21) with the value of V_{rs} computed with Eq. (6) of the paper indeed yields excellent agreement with measurements of V_s reported in Ref. [4]. Moreover, the best fit of Eq. (21) to the experimental data gives $a \approx 4.6$ which is close to the value $a \approx 4.4$ reported in Ref. [4] using the approximated expression for V_{rs} valid for large grain sizes.

III. ESTIMATING THE SATURATION LENGTH L_s FROM THE SIZE OF DUNES

The saturation length of sediment transport can be estimated indirectly from the scale of dunes formed in a given environment. Specifically, two methods can be used.

In the first method, described in Ref. [18], L_s is determined from the minimal cross-stream width of barchan dunes occurring in a given environment. As a matter of fact, barchans which are smaller than a minimal size do not display limbs or slip face and are called domes. The largest dome which has neither slip face nor limbs (and is smaller than the smallest barchan dune with slip face in the field) indicates the minimal dune size in the field [18]. The cross-stream width of the minimal dune scales with the saturation length as, $W \approx 12 L_s$ [18].

The second method consists of extracting L_s from the wavelength (crest-to-crest distance) of the so-called “elementary dunes” [19]. Examples of elementary dunes are those smallest superimposed bedforms occurring on a flat surface or on top of a large barchan dune [20, 21], which may form, for instance, due to a storm wind that makes a small angle with the dominant transport direction [20].

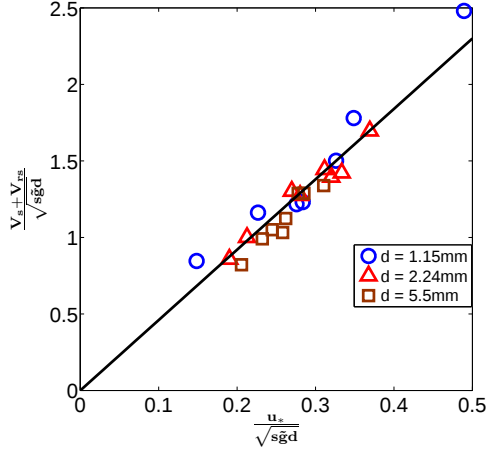


Fig. S 1. Average value of the dimensionless fluid speed $\frac{U_s}{(s-1)gd} = \frac{V_s + V_{rs}}{(s-1)gd}$, as a function of the dimensionless shear velocity $\frac{u_*}{(s-1)gd}$. For the symbols, the average dimensionless particle speed $\frac{V_s}{(s-1)gd}$ was obtained from measurements [4], while we computed V_{rs} using Eq. 6 of the paper with $\mu = 0.5$. The black solid line corresponds to the best fit to the experimental data using Eq. (21), which yields $a \approx 4.6$.

In the following section, we present a summary of the method used to obtain L_s from the wavelength of elementary dunes. In the subsequent sections, we use this method and the method of the cross-stream width of the

minimal barchan in order to estimate L_s on Earth, Mars, Venus, and under water.

A. How to estimate L_s from the wavelength of elementary dunes occurring on the surface of a sediment bed

In this section, we describe how to obtain the saturation length from the wavelength λ of the so-called “elementary dunes” by using the method described in Fourrière *et al.* [19]. These authors have shown how to compute the spatial shear stress $\tau(x)$ on top of the transport layer (or, in the absence of transport, on top of the sediment bed) for a flat sediment bed which has a small perturbation $h(x)$ in the vertical direction. Let τ_o denote the undisturbed shear stress corresponding to $h(x) = 0$. Then, the Fourier-transformed shear stress, $\hat{\tau}(k)$, can be written as [19],

$$\hat{\tau} = \tau_o(A + iB)k\hat{h}, \quad (22)$$

where k is the wavenumber and the “hat” denotes that the Fourier-transformed value of the corresponding quantity is considered. Fourrière *et al.* [19] used a turbulence model to compute A and B (which are both positive numbers) as functions of k and z_o^* (the apparent roughness), which is the surface roughness z_o modified due to the presence of the transport layer. The numerical results obtained by the authors can be fitted to [19],

$$A(R) = 2 + \frac{1.0702 + 0.093069R + 0.10838R^2 + 0.024835R^3}{1 + 0.041603R^2 + 0.0010625R^4}, \quad (23)$$

$$B(R) = \frac{0.036989 + 0.15765R + 0.11518R^2 + 0.0020249R^3}{1 + 0.0028725R^2 + 0.00053483R^4}, \quad (24)$$

where $R = \ln \frac{2\pi}{kz_o^*}$. The wavelength $\lambda = 2\pi/k_{\max}$ of the elementary dunes corresponds to the wavenumber $k_{\max}$ under which the dunes grow fastest [19]. By using instability analysis, Fourrière *et al.* [19] showed that λ is related to the saturation length L_s through the equation,

$$\frac{2\pi L_s}{\lambda} = X^{-1/3} + X^{1/3}, \quad (25)$$

where the quantity X is defined as,

$$X = -\frac{\tilde{B}}{\tilde{A}} + \sqrt{1 + \frac{\tilde{B}^2}{\tilde{A}^2}}, \quad (26)$$

while $\tilde{A}$ and $\tilde{B}$ incorporate dependence on the fluid shear velocity (u_*),

$$\tilde{A} = A(R_{\max}) - \frac{\gamma_c A(R_{\max})}{1 - \gamma_c} \frac{u_t^2}{u_*^2}, \quad (27)$$

$$\tilde{B} = B(R_{\max}) - \frac{\gamma_c B(R_{\max}) + \mu_c^{-1}}{1 - \gamma_c} \frac{u_t^2}{u_*^2}. \quad (28)$$

In the equations above, $R_{\max} = \ln \frac{2\pi}{k_{\max} z_o^*} = \ln \frac{\lambda}{z_o^*}$, $\mu_c \approx \tan(32^\circ)$ is the dynamic angle of repose of the sand, and $\gamma_c \approx 0.5$ and 0. for the subaqueous and aeolian transport regimes, respectively [19].

In order to estimate L_s using the method described above, knowledge of the threshold shear velocity for sustained sediment transport, u_t , as well as the apparent roughness, z_o^* , is required. For both u_t and z_o^* , the equation adopted depends on the transport regime. For

the subaqueous regime, u_t is computed from the Shields curve [22]. Furthermore, it is known that z_o^* does not vary strongly with flow conditions [23], and we thus approximate z_o^* as the surface roughness in the absence of transport (z_o). For the aeolian regime, u_t and z_o^* are calculated using the analytical model derived in Ref. [2].

We emphasize that the analytical models involved in the estimation of L_s using the method described above potentially introduce significant systematic errors, which could possibly obscure any trend in the data, and which make these estimations of L_s particularly uncertain.

B. Earth

In Fig. 1 of the paper, we show experimental data corresponding to measurements of the saturation length of aeolian sediment transport under Earth conditions. These measurements are described in the paragraphs which follow.

Andreotti et al. [24] performed wind-tunnel measurements of the saturation length in aeolian transport under Earth conditions using quartz sand ($\rho_p = 2650 \text{ kg/m}^3$) of average diameter $d = 120 \mu\text{m}$. The authors measured the sediment flux profiles $Q(x)$ of particles transported over a flat sand bed in the wind tunnel. The flux profiles $Q(x)$ measured at downstream positions (x) where the condition $Q(x) > 0.8Q_s$ was fulfilled (i.e. $|1 - Q/Q_s| \ll 1$) were fitted using the equation,

$$Q(x) = Q_s \cdot \left[1 - \exp\left(-\frac{x - x_o}{L_s}\right) \right], \quad (29)$$

whereby L_s and x_o were used as fit parameters [24]. The fitted value L_s is the saturation length, since Eq. (29) is a solution of Eq. (1) of the Letter for a flat sand bed ($Q_s(x) = Q_s$). The values of L_s obtained in Ref. [24] in this manner correspond to the brown squares in Fig. 1 of the paper. The error bars associated with the measurements [24] are also displayed in the figure.

Furthermore, indirect estimates of L_s of aeolian transport on Earth were obtained from the wavelength λ of elementary dunes on Earth's dune field by using the method described in Section III A [24]. The sand of the dunes considered in the measurements consisted of quartz particles ($\rho_p = 2650 \text{ kg/m}^3$) of average diameter $d = 185 \mu\text{m}$ [24]. The data corresponding to these estimates are denoted by the green circles in Fig. 1, whereby the error bars denote uncertainties in the measurement of λ as described by Andreotti et al. [24]. Note that, instead of using the method of Ref. [2], the authors estimated z_o^* from empirical fits to the data of Ref. [25]. However, this slightly different method yields a value of z_o^* that is very similar to the one obtained with the equations presented in Ref. [2]. Consequently, approximately the same value of L_s is obtained using either method for estimating z_o^* , since the equations presented in Ref. [2] have been shown to agree very well with the data of Ref. [25].

C. Mars

We estimate the saturation length under Martian conditions from the cross-stream width W of the minimal dune. Ref. [26] reported values of W for two different barchan dune fields on Mars, namely $W \approx 200 \text{ m}$ for the barchan dune field in Arkhangelsky crater, which is located in the southern highlands of Mars, and $W \approx 80 \text{ m}$ for another dune field, which is located near the north pole [26]. From these values of the minimal cross-stream width, we obtain $L_s \approx 16.7 \text{ m}$ for the Arkhangelsky barchan dune field and $L_s \approx 6.7 \text{ m}$ for the dune field near the north polar region from $W \approx 12 L_s$ [26].

Furthermore, in order to predict L_s for each dune field using Eq. (10) of the Letter, we need to estimate average density and viscosity of the local atmosphere. Average surface pressure (P) and temperature (T) values for both fields were obtained from the v23 ARC Mars GCM (see Refs. [27–29] for details), yielding $P = 540 \text{ Pa}$, $T = 201 \text{ K}$, and $P = 811 \text{ Pa}$, $T = 165 \text{ K}$ for the Arkhangelsky crater and north pole dune fields, respectively. From these estimates, we obtain the following average values of fluid viscosity and density: $\nu = 7.23 \times 10^{-4} \text{ m}^2/\text{s}$ and $\rho_f = 0.0141 \text{ kg/m}^3$ for the Arkhangelsky crater dune field, and $\nu = 3.11 \times 10^{-4} \text{ m}^2/\text{s}$ and $\rho_f = 0.026 \text{ kg/m}^3$ for the dune field near the north pole. On the other hand, both the particle size d of Martian dunes and the typical shear velocity u_{typ} for which the dune fields were formed are poorly known [8]. As described in the main article, we therefore calculate L_s for a range of particle sizes d ($100 - 600 \mu\text{m}$ [30]) and for two estimates of u_{typ} . The first estimate uses $u_{\text{typ}} = u_{\text{ft}}$, consistent with previous studies [18, 31–33], and we derive the second estimate of u_{typ} below, using the wind speed probability distribution measured at the Viking 2 landing site [34, 35].

1. Estimating u_{typ}

In the following, we estimate u_{typ} by first obtaining the probability distribution of wind shear velocities at the Viking 2 lander site from measurements [34, 35]. Due to the scarcity of wind speed measurements on the red planet, we then assume out of necessity that this wind shear velocity probability distribution occurs at the Arkhangelsky and north pole dune fields. Using expressions for (i) the saturated mass flux of saltating particles, Q_s [2], as a function of u_* and (ii) the probability P_{tr} that saltation occurs when $u_t < u_* < u_{\text{ft}}$, we obtain estimates of u_{typ} .

We start our approach by defining u_{typ} as the saltation-flux weighted average of u_* . That is,

$$u_{\text{typ}} = \frac{\int_0^\infty u_* P_{u_*} P_{\text{tr}}(u_*) Q_s(u_*) du_*}{\int_0^\infty P_{u_*} P_{\text{tr}} Q_s(u_*) du_*}, \quad (30)$$

where P_{u_*} is the probability of occurrence of winds with shear velocity u_* , P_{tr} is the probability that saltation is occurring for a given u_* , and Q_s is the equilibrium sand transport rate (provided saltation is occurring) for a given value of u_* . We derive expressions for each of these functions below.

2. The probability distribution of the wind shear velocity

The probability distribution of wind speeds is commonly described using the Weibull distribution [36], given by,

$$P(U) = \frac{k}{c} \cdot \left(\frac{U}{c}\right)^{k-1} \cdot \exp\left[-\left(\frac{U}{c}\right)^k\right], \quad (31)$$

where U is the wind speed at a given height z , k is a dimensionless shape parameter, and c is a scale speed that is proportional to the average wind speed through,

$$c = \frac{\bar{U}}{\Gamma(1 + 1/k)}. \quad (32)$$

Here, $\bar{U}$ is the wind speed averaged over much longer time periods than U ; typically, U is averaged over several minutes to an hour, and $\bar{U}$ is averaged over one or several years. The long-term averaged $\bar{U}$ can be related to the long-term averaged shear velocity $\bar{u}_*$ through the “law of the wall” [37]. That is,

$$\bar{U} = \frac{\bar{u}_*}{\kappa} \ln\left(\frac{z}{z_0}\right), \quad (33)$$

where z_0 is the aerodynamic roughness length, and $\kappa \approx 0.40$ is the von Kármán constant. Combining the above

equations then yields the probability distribution of u_* ,

$$P(u_*) = \frac{k}{c_{u_*}} \cdot \left(\frac{u_*}{c_{u_*}}\right)^{k-1} \cdot \exp\left[-\left(\frac{u_*}{c_{u_*}}\right)^k\right], \quad (34)$$

where,

$$c_{u_*} = \frac{\kappa c}{\ln(z/z_0)}. \quad (35)$$

We obtain the wind speed scaling parameters c and k from the only long-term data set of Martian wind speeds: the measurements made by the Viking 2 lander over a period of 1040 sols [34]. We thus use $c = 3.85$ m/s and $k = 1.22$ m/s as calculated by Ref. [35]. Moreover, we use $z_0 = 1$ cm after Ref. [38].

3. The saltation transport probability P_{tr}

Predicting the sand transport rate for a given value of u_* is complicated by the occurrence of hysteresis in Martian saltation [8, 39]. That is, recent studies have estimated that the value of u_* for which saltation is initiated (u_{ft}) exceeds the minimum value of u_* for which saltation can be sustained (u_t) by up to a factor of ~ 10 [8, 39]. Whether or not saltation is occurring for a given value of u_* intermediate between u_t and u_{ft} thus depends on whether the wind speed exceeded u_{ft} more recently than that it dropped below u_t . By also assuming that the probability distribution of Martian wind speeds can be described by a Weibull distribution, Ref. [39] derived an equation estimating the probability that transport occurs for values of u_* intermediate between u_t and u_{ft} ,

$$P_{tr} = \frac{\exp\left\{-[u_t \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\} - \exp\left\{-[u_{ft} \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\}}{1 - \exp\left\{-[u_t \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\} + \exp\left\{-[u_{ft} \cdot \Gamma(1 + 1/k)/\bar{u}_*]^k\right\}}, \quad (u_t < u_* < u_{ft}), \quad (36)$$

where u_{ft} is calculated following Ref. [40]. Of course, when $u_* < u_t$ and $u_* > u_{ft}$, we have that,

$$P_{tr} = 0, \quad (u_* < u_t), \quad (37)$$

$$P_{tr} = 1, \quad (u_* > u_{ft}). \quad (38)$$

4. The estimated u_{*typ} and L_s

We can now estimate u_{*typ} by calculating the mass flux Q_s using Eq. (69) in Ref. [2], and inserting this together with Eqs. (34)–(38) into Eq. (30). The resulting estimate of u_{*typ} is plotted in Fig. S 2a as a function of particle

size at both dune fields. Since Martian wind speeds rarely exceed u_{ft} , at least at the Viking 2 landing site [34, 35], we find that u_{*typ} is closer to u_t than to u_{ft} . The resulting saturation length L_s predicted with these values of u_{*typ} and Eq. (2) of the main article is plotted in Fig. S 2b.

D. Subaqueous dunes

By using the method described in Section III A, Fourrière et al. [19] estimated the saturation length of subaqueous transport from measurements of the wavelength λ of elementary transverse bedforms produced in

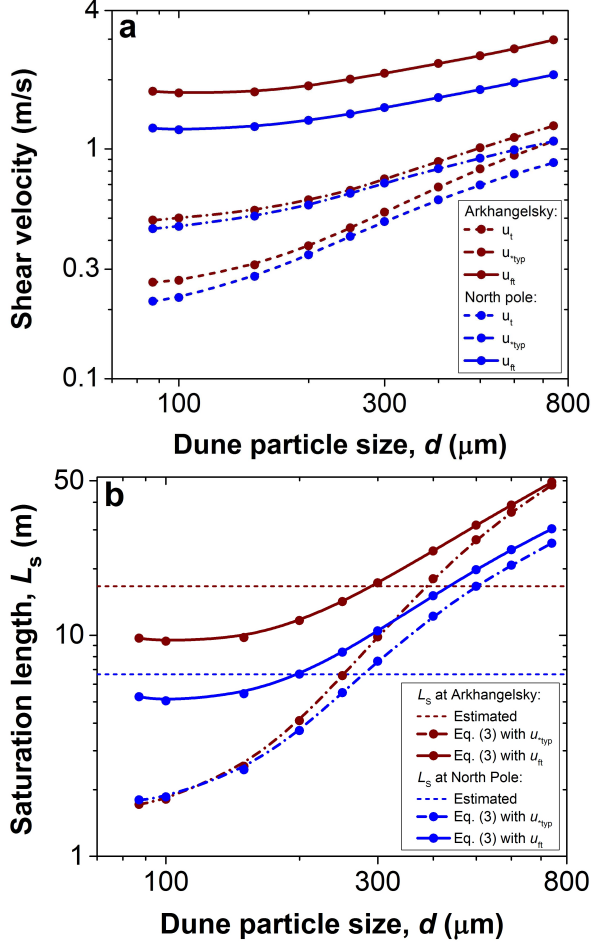


Fig. S 2. (a) The estimated typical shear velocity u_{*typ} for which bedforms are formed on Mars (dash-dotted lines) for the dune fields at Arkhangelsky crater (brown lines) and the north pole (blue lines). Also shown are the fluid threshold u_{ft} at which sediment transport is initiated (solid lines) and the impact threshold u_t below which transport cannot be sustained (dashed lines). (b) Saturation length predicted with Eq. (2) and u_{*typ} derived from Eq. (30) (dash-dotted lines) and from $u_{*typ} = u_{ft}$ (solid lines). Also shown are the values of L_s estimated from the minimal size of barchan dunes (dashed lines).

water flumes [41–43]. These experiments were performed using different types of granular materials, namely natural sand particles or glass beads. The data corresponding to the estimates of L_s by Fourrière et al. from these experiments are represented by the blue (for natural sand particles) and green (for glass beads) symbols in Fig. 2 of the paper.

Furthermore, we have estimated L_s for subaqueous transport from the minimal cross-stream width W of subaqueous barchans produced in the experiments by

Franklin and Charru [44]. From these measurements, we obtain L_s from the relation, $L_s = W/12$ [18]. The red symbols in Fig. 2 denote the values of L_s estimated in this manner.

E. Venus

We estimate the saturation length of sediment transport on Venus from the wavelength of microdunes produced in wind tunnel experiments by Marshall and Greeley [45] mimicking the Venusian atmosphere.

By adjusting the pressure and the temperature in the wind tunnel, Marshall and Greeley [45] obtained values of air viscosity ($\nu = 2.9 \times 10^{-7} \text{ m}^2/\text{s}$) and air density ($\rho_f = 52.94 \text{ kg/m}^3$) similar to those occurring in the atmosphere of Venus. Experiments using natural sand (quartz particles) of average diameter $d = 150 \mu\text{m}$ under free stream velocity $U_\infty \approx 0.8 \text{ m/s}$ produced microdunes of wavelength $\lambda \approx 20 \text{ cm}$ (cf. Fig. 7 of Ref. [45]). From Fig. 2 of Ref. [45] we obtain the threshold free stream velocity $U_{\infty t}$ corresponding to the grain diameter $d = 150 \mu\text{m}$ used in the experiments, that is, $U_{\infty t} \approx 0.6 \text{ m/s}$. From the value of $U_{\infty t}$, we can estimate the shear velocity ratio u_*/u_t in leading order, $u_*/u_t \approx U_\infty/U_{\infty t} \approx 1.3$. By taking this value of u_*/u_t and using the method described in Section III A, we estimate $L_s \approx 6 \text{ mm}$ from the wavelength of the Venusian microdunes produced in the wind tunnel. We display this indirect estimate of L_s in Fig. 3 of the paper.

IV. A SIMPLE EXPRESSION TO COMPUTE L_s IN THE AEOLIAN REGIME

In this Section, we present a simplified version of our expression for the saturation length L_s of sediment transport in the aeolian regime, that is, Eq. (10) of the paper, which is valid for large values of u_*/u_t .

We note that the quantity K , given by Eq. (9) of the paper, can be approximated as its limit for large dimensionless shear velocities ($(u_*/u_t)^2 \gg 1$), giving $K \approx V_s/(FV_{rs})$. This approximation yields the simple expression,

$$L_s^{\text{aeolian}} \approx 3c_v V_s^2 \cdot [\mu g]^{-1}. \quad (39)$$

The advantage of this approximation is that it provides a simpler expression for computing L_s as a function of the steady-state particle velocity V_s in the aeolian regime. In this section we discuss the performance of this approximated expression in comparison to the original expression ((Eq. 10) of the paper).

The approximated expression (Eq. (39)), resulted from approximating the feedback term K (Eq. (9) of the paper) as its limit for large dimensionless shear velocities ($(u_*/u_t)^2 \gg 1$). Consequently, the approximation performs best in the range of large values of the ratio u_*/u_t , as we can see in Fig. 3. This figure shows $L_s/(sd)$ as

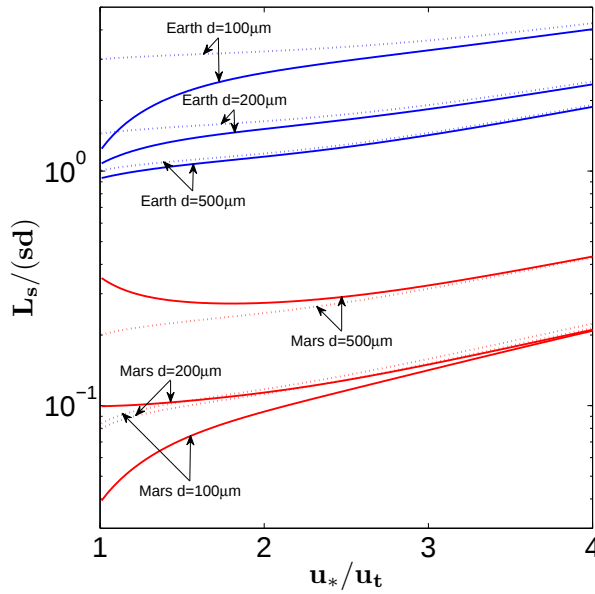


Fig. S 3. Dimensionless saturation length, $L_s/(sd)$, as a function of the dimensionless shear stress, u_*/u_t for aeolian transport. Shown are predictions with different values of d for Earth (blue lines) and Mars (red lines) conditions using the original expression for L_s in the aeolian regime (Eq. (10) of the paper; continuous lines) and the approximated expression (Eq. (39); dotted lines).

a function of u_*/u_t and different values of d for Earth (blue lines) and Mars (red lines) conditions using the original expression for L_s in the aeolian regime (Eq. (10) of the paper; continuous lines) and the approximated expression (Eq. (39); dotted lines). We note that for all conditions, the deviation increases as u_* approaches u_t . Indeed, we see in Fig. 3 that, for typical conditions where the particle size is between $200\mu\text{m}$ and $500\mu\text{m}$ [8, 30, 46], the predictions of L_s from both expressions, original and approximated, deviate from each other by less than 30%.

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