FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

ENGINEERING
OF ADVANCED
MATERIALS

INSTITUTE FOR
MULTISCALE SIMULATION
OF PARTICULATE SYSTEMS

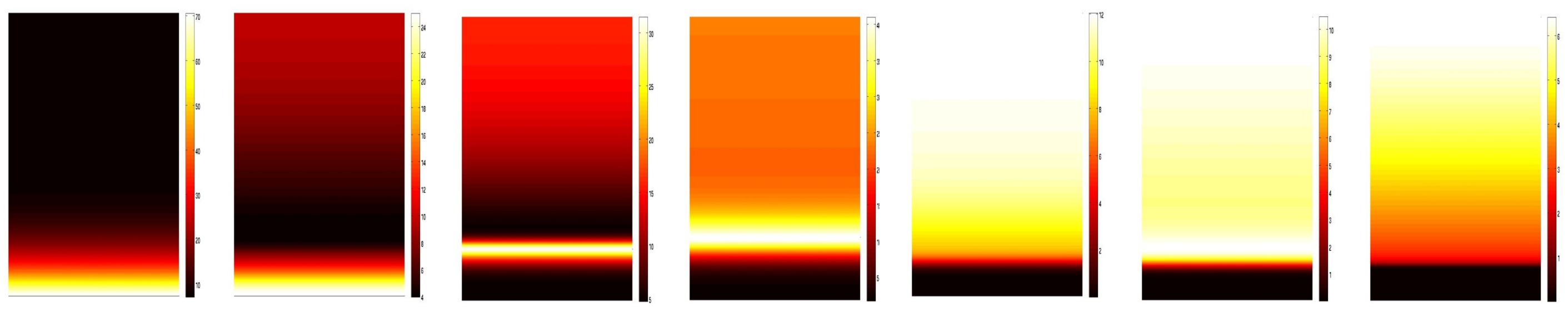
Sedimentation and Collapse of a granular gas under gravity

Lídia Almazán[1], Dan Serero[1], Thorsten Pöschel[1] & Clara Salueña[2]

[1] Institute for Multiscale Simulation, Engineering of Advanced Materials, Friedrich-Alexander-Universität Erlangen-Nürnberg
[2] Departament d'Enginyeria Mecànica, Universitat Rovira i Virgili, 43007 Tarragona, Spain

Goal and Motivation

The sedimentation and collapse of a granular gas under gravity represents a simple and experimentally feasible example of inelastic collapse under external forcing. Despite its apparent simplicity, such a system exhibits a rich temporal behavior, and its cooling properties have recently been the object of contradictory reports [1,2,3]. We present a hydrodynamic study of the gravity driven collapse of a granular gas initially heated from below, when the energy supply is switched off.



Successive snapshots of the temperature field during the sedimentation of a granular column under gravity after the initial bottom heating is switched off.

Model description

We consider a granular fluid composed of smooth inelastic hard disks whose collisions are characterized by a fixed coefficient of restitution. We solve numerically the hydrodynamic equations for a 2D gas using the Jenkins-Richman transport coefficients [6]:

$$\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{u})=0,$$

$$n\left(\frac{\partial \vec{u}}{\partial t}+\vec{u}\cdot\vec{\nabla}\vec{u}\right)=-\vec{\nabla}\cdot\hat{P}-n\vec{g},$$

$$n\left(\frac{\partial T}{\partial t}+\vec{u}\cdot\vec{\nabla}T\right)=-\nabla\cdot\vec{q}-\hat{P}:\vec{\nabla}\vec{u}-\xi nT$$

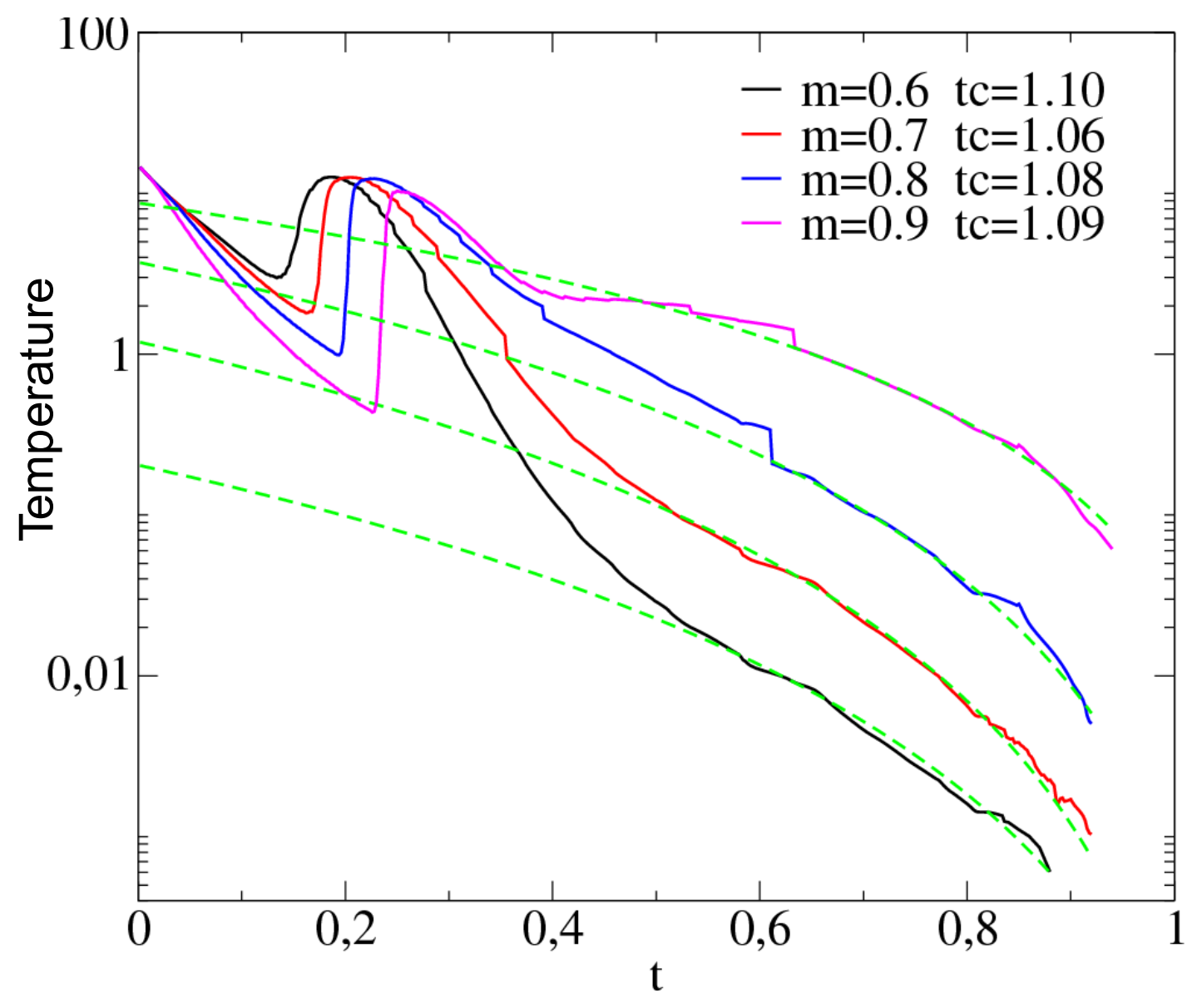
The Navier-Stokes terms are treated by centered high-order explicit in time finite difference approximations and considered as sources for the method of lines in the time approximation. The Euler terms are solved in local coordinates by a fifth-order explicit in time finite difference characteristic-wise WENO method [4].

Late stage of cooling and collapse

Close to collapse, when all the material is densely packed on the bottom and almost at rest, the system exhibit a scaling behavior where the (kinetic) energy decays according to a power law:

$$E \sim (t_c - t)^{\beta}$$

but contradictory results were reported concerning the value of β [1,2,3]. In order to estimate β, the late stages of the energy (or temperature) decay were fitted with the above power law, with parameters β and t_c.

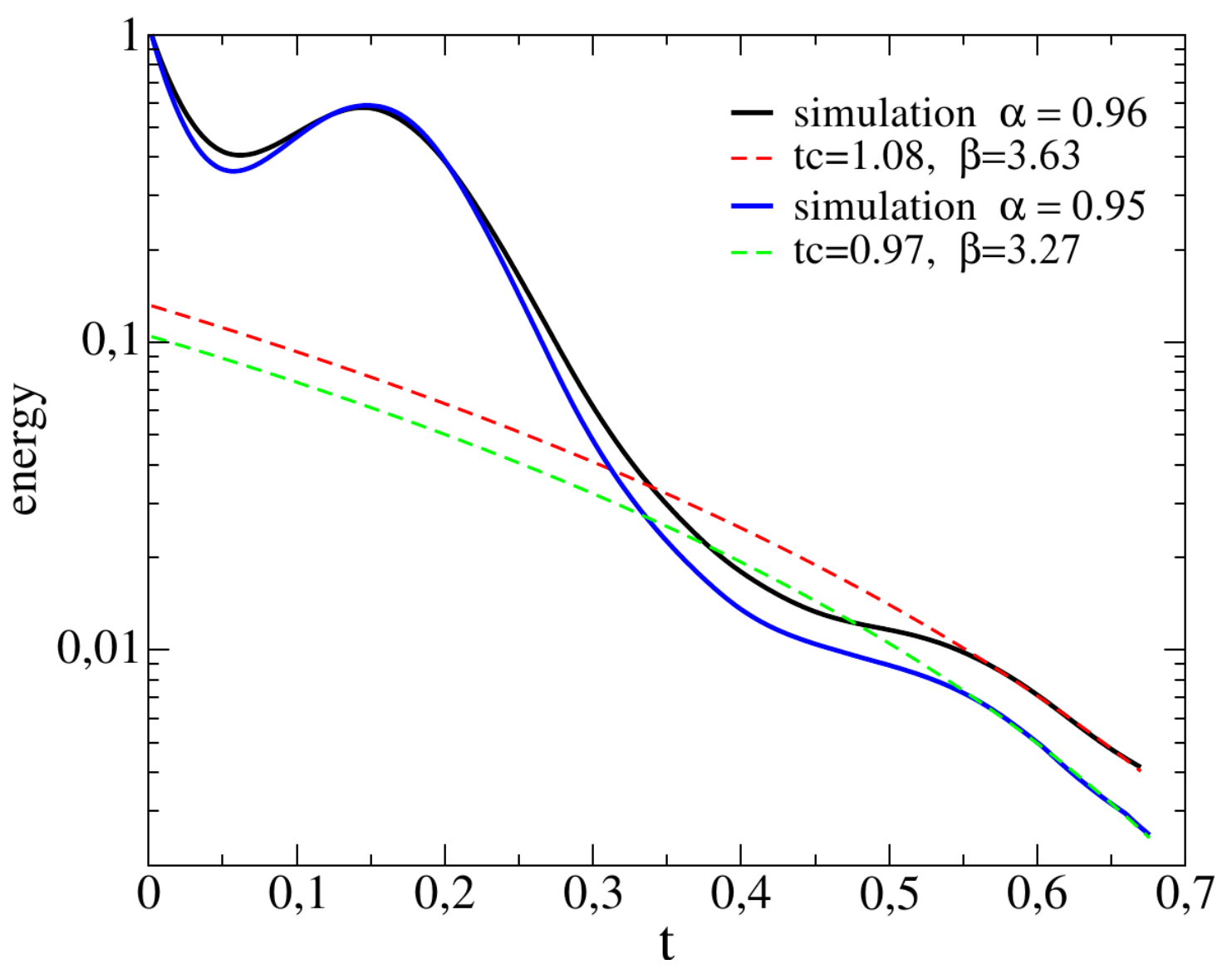


Considering the temperature of various layers of the material, characterized by different Lagrangian (mass) coordinates, very similar values for t_c are obtained. This confirms the existence of a well defined collapse time, and imply:

$$T(m,t)=Q(m)(t_c-t)^{\beta(m)}$$

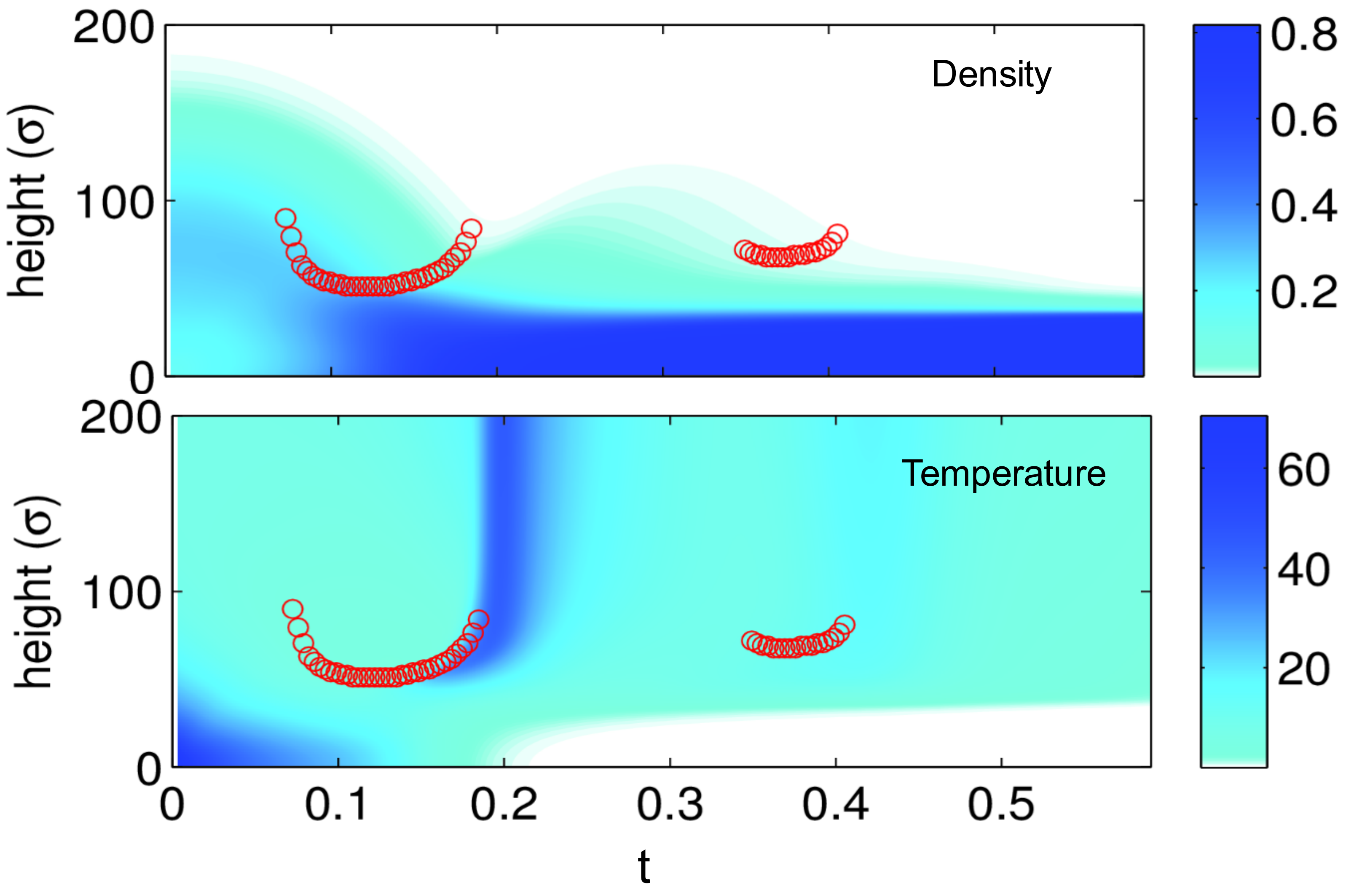
in accordance with previous numerical (MD) [3] and experimental [1] findings.

Considering the overall kinetic energy, it is found that β depends on the values of the coefficient of restitution α.
E.g, for $\alpha = 0.96$, $3.5 < \beta < 5.2$.
The values are consistent with experimental findings [1], as well as with results of force based MD simulations [2] (e.g. $3.8 < \beta < 4.2$ for $\alpha = 0.96$), but contradicts the claim in [3] that $\beta = 2$.



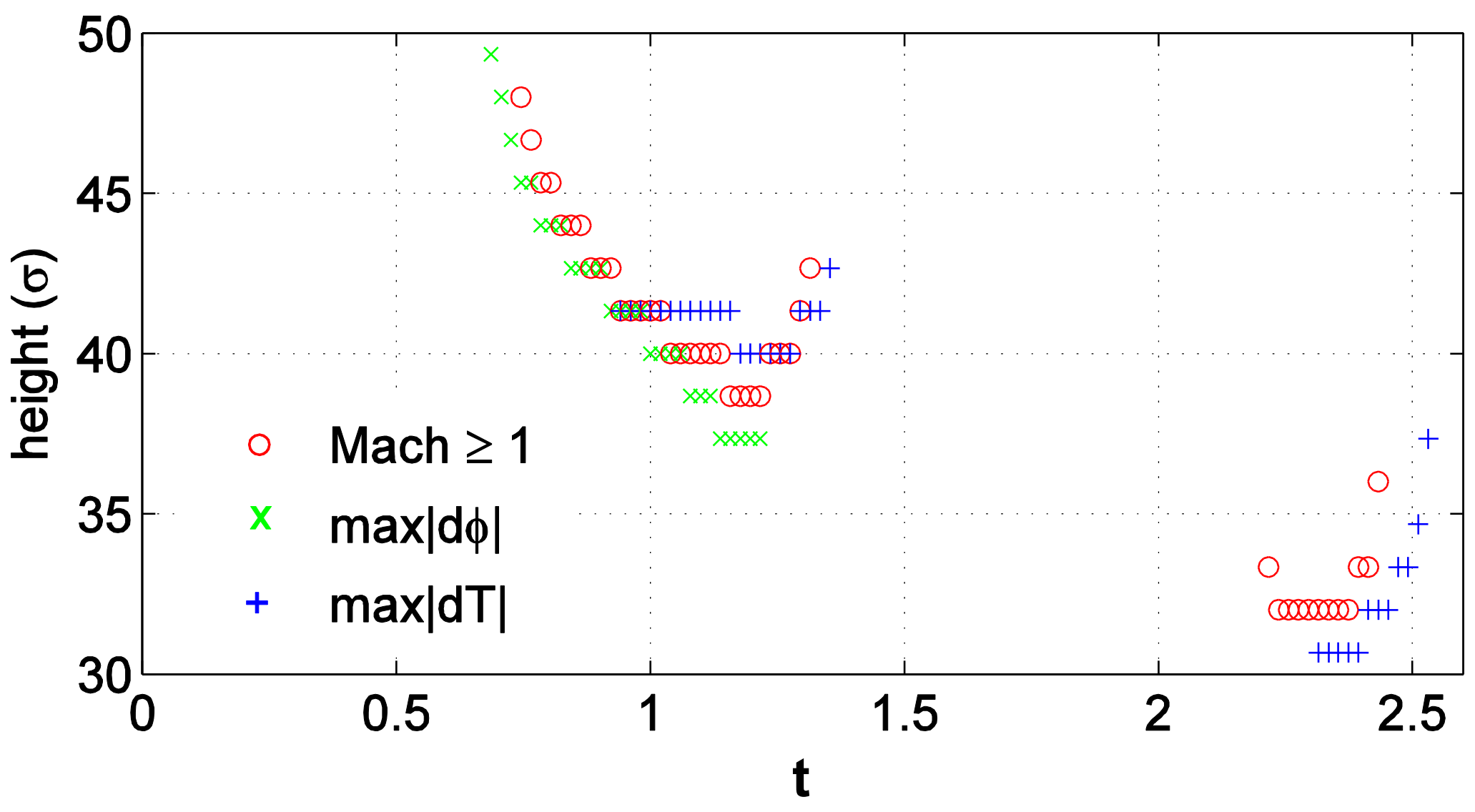
Shock waves

The hydrodynamic simulation starts with the initial condition corresponding to a system heated from below by a thermal wall at fixed temperature [5]. During the evolution, successive shocks are seen to develop and propagate, accompanied with steep temperature and density fronts. Simulations performed for various initial bottom plate temperature T_0 and different coefficients of restitution α yields similar behaviors. For $\alpha < 0.95$ only one shock is observed.



Evolution of the temperature and density fields for $\alpha = 0.98$. Heights are measured in units of particle diameters.

The separation between subsonic and supersonic regions (Mach=1), represented by red circles, coincide with the temperature and density fronts, characterized by the maximum value of their gradients.



Conclusions

- The sedimentation process presents a rich behavior, comprising several stages of diffusive (subsonic) and inertial (supersonic) dynamics.
- Shock waves develop and propagate, followed by expansion of the material and sharp increase of temperature.
- The front separating between subsonic and supersonic regimes follows the sharp profiles of temperature and density fields.
- Close to collapse, the energy decays according to a power law, with a well defined collapse time.
- The power laws describing the energy decay are found to match the experimental values, as well as the predictions of force-based Molecular Dynamics simulations, and contradict the claim of a universal value for β made in early analytical studies.

References

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