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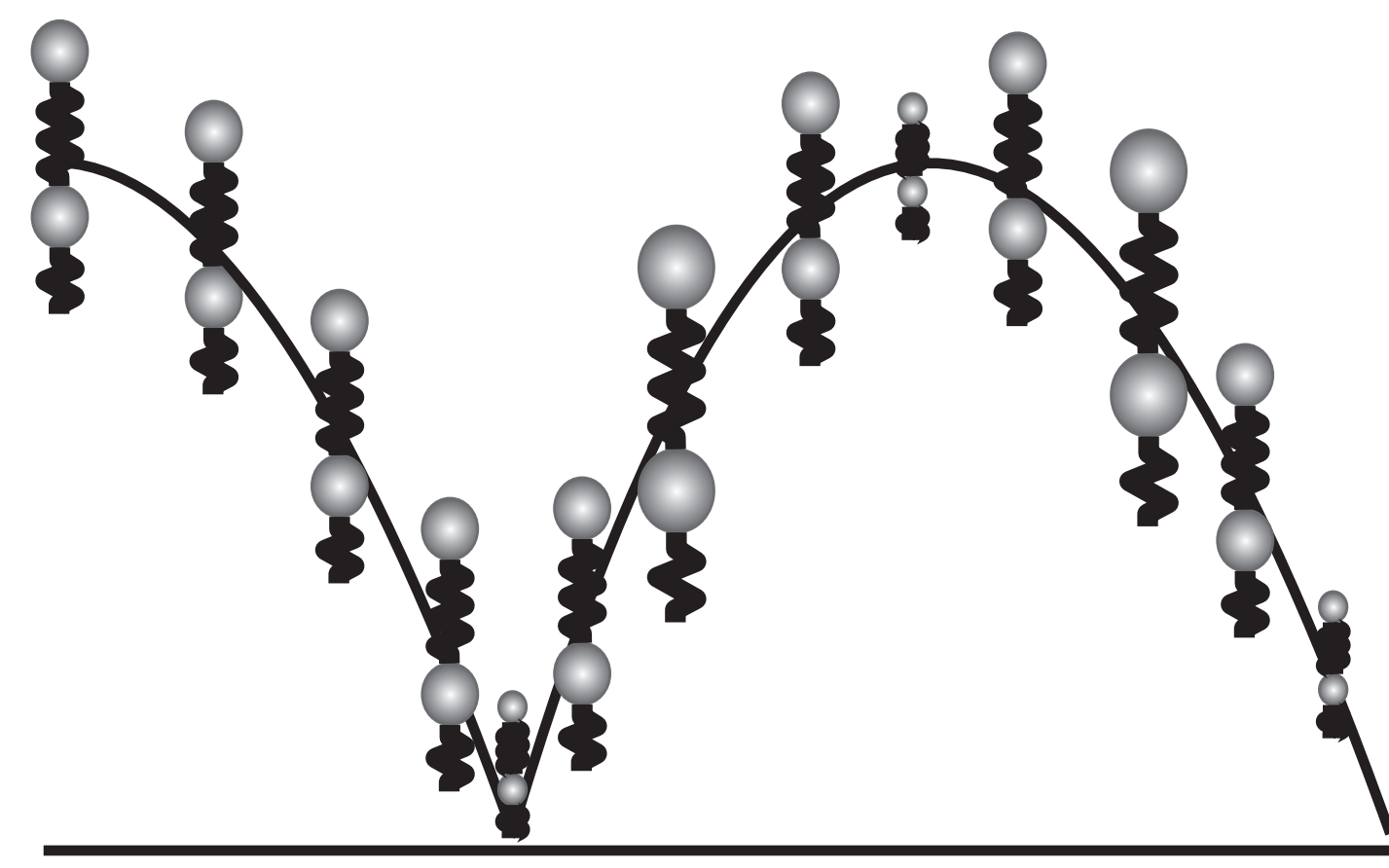
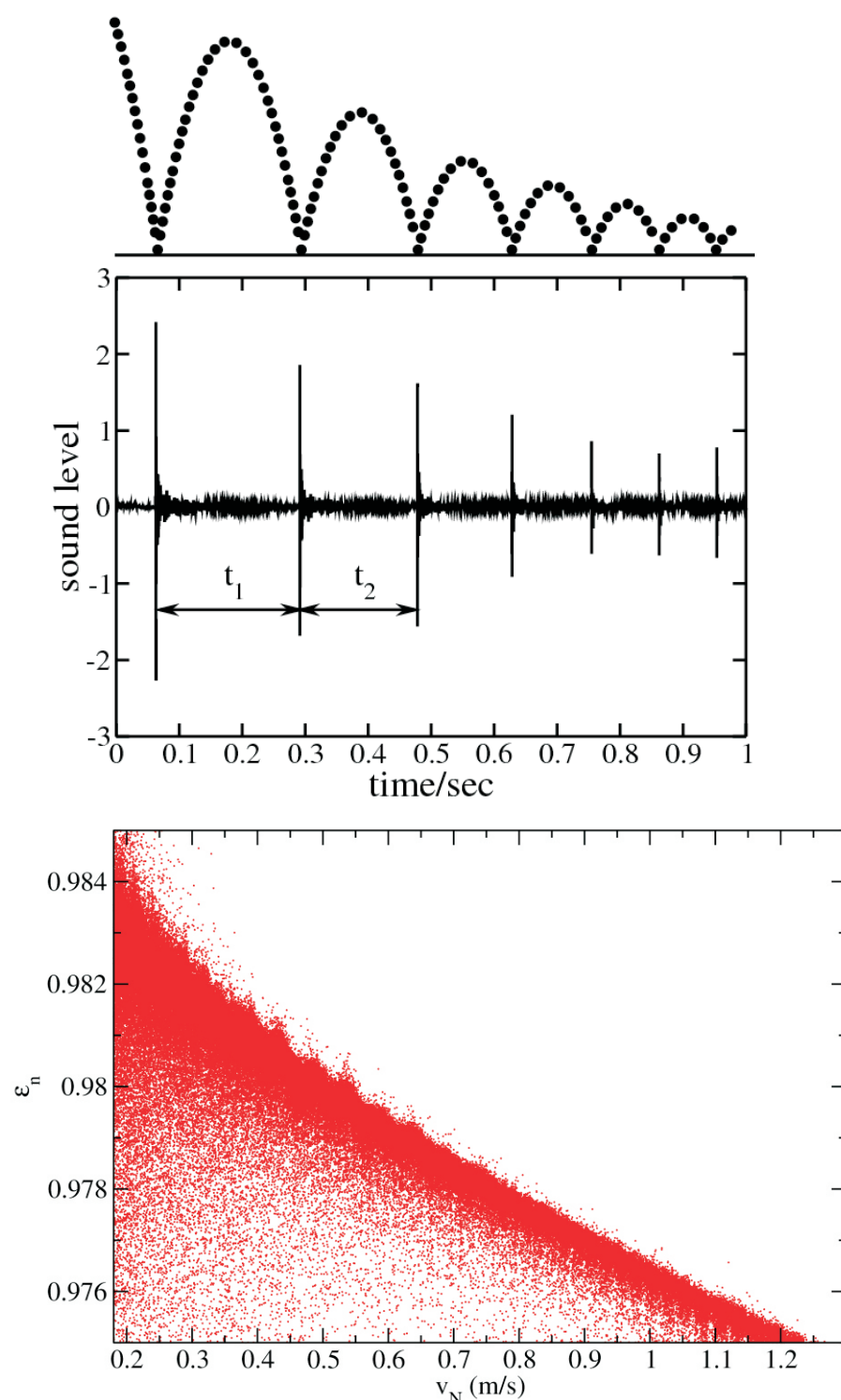
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Setup

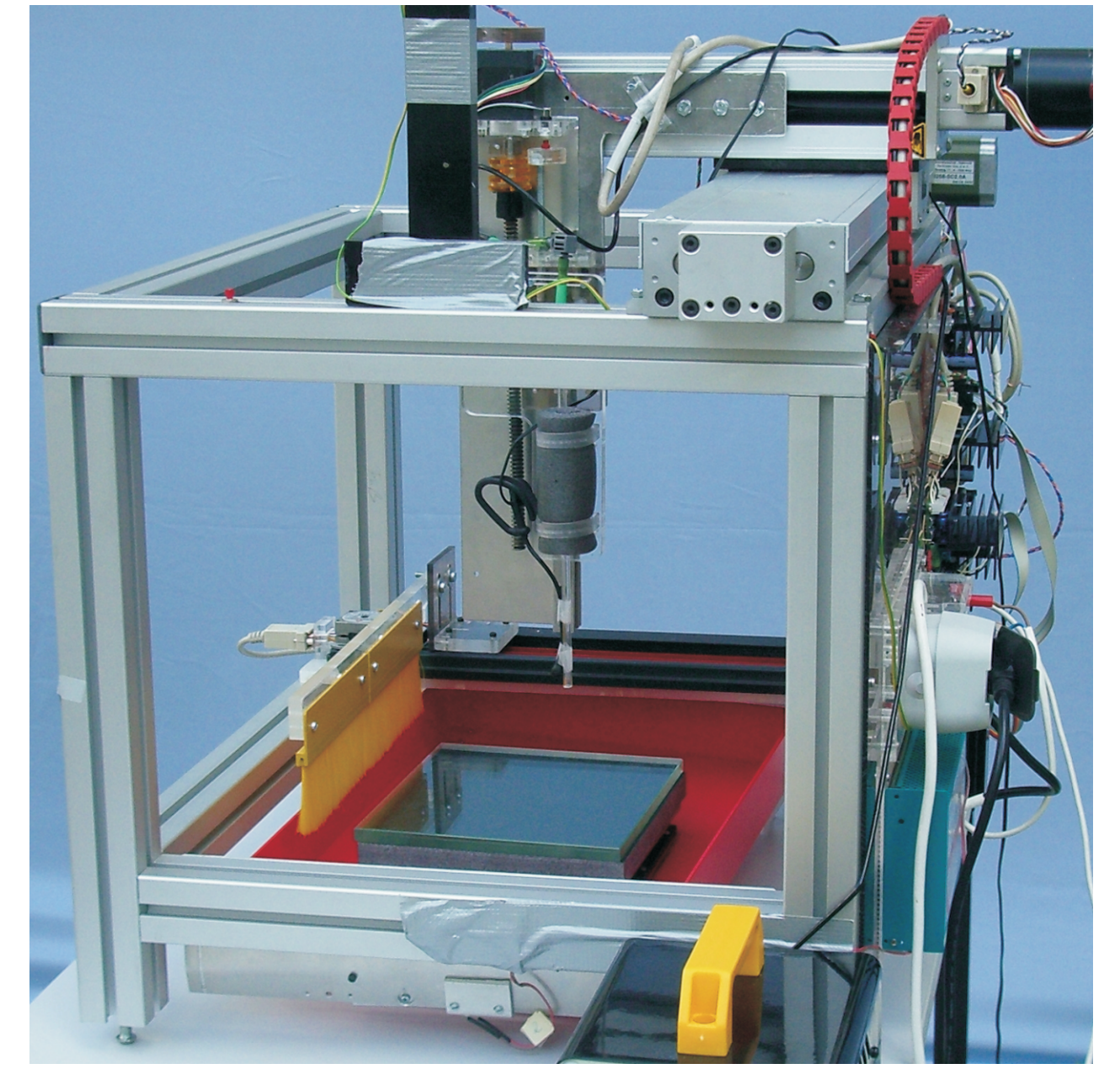
We are measuring the motion of a jumping ball using a robot which records the sound caused by a steel sphere hitting a plate. The degree of energy loss, i.e. the dissipative properties, are characterized by the coefficient of restitution. It is defined as the ratio between the post-collisional and the pre-collisional velocities.

Three impacts are needed to calculate the coefficient of normal restitution $\epsilon(v_i) = v_2/v_1 = t_2/t_1$. We are using several plate materials (glass, liquid metal, SiC* and lexan) to measure their influence on the coefficient of restitution. We are also analyzing the distribution of ϵ in the measurements.

* provided by Matthias Schröter
* provided by Peter Wellmann



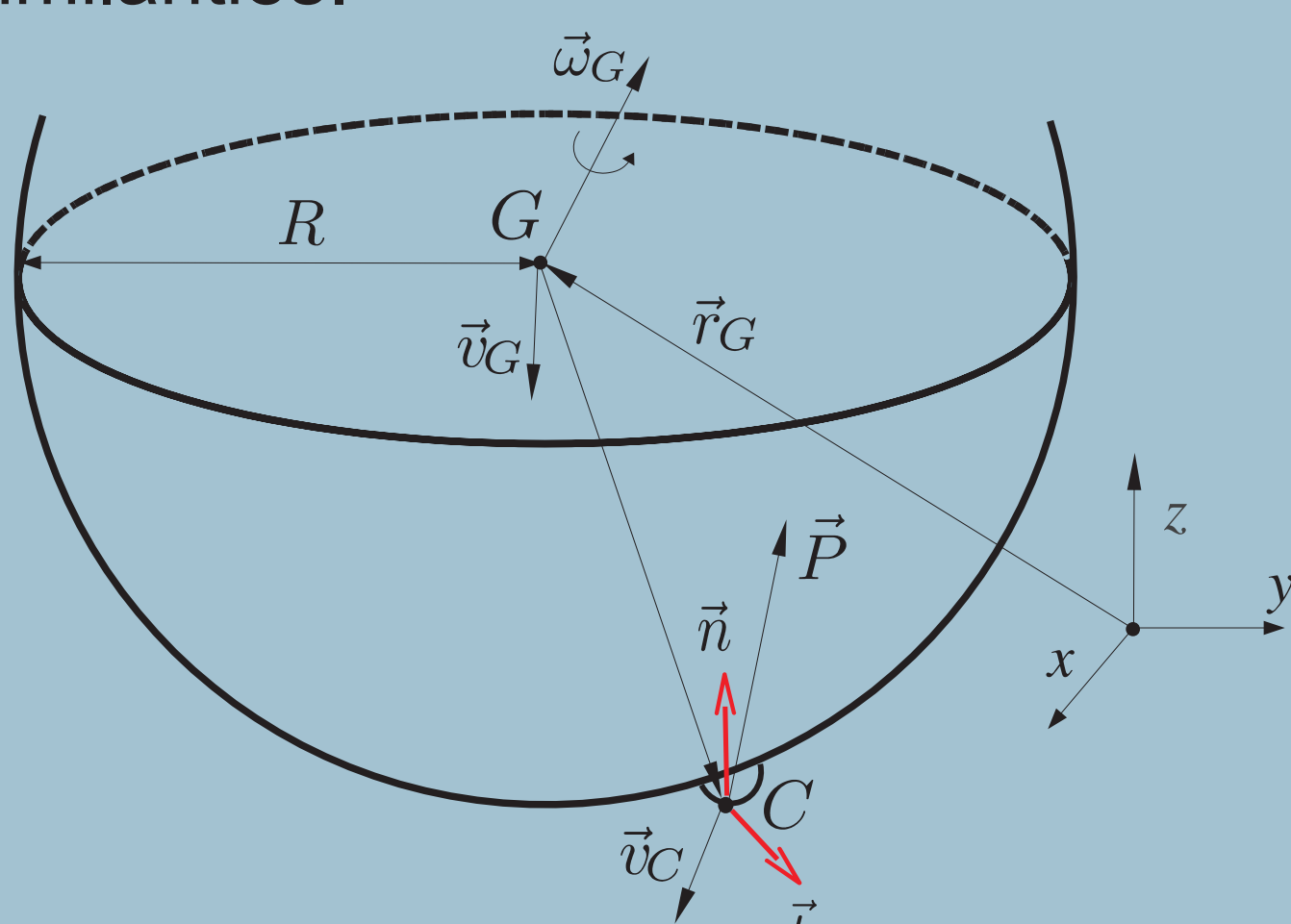
Model of an oscillating object bouncing on a rigid surface.



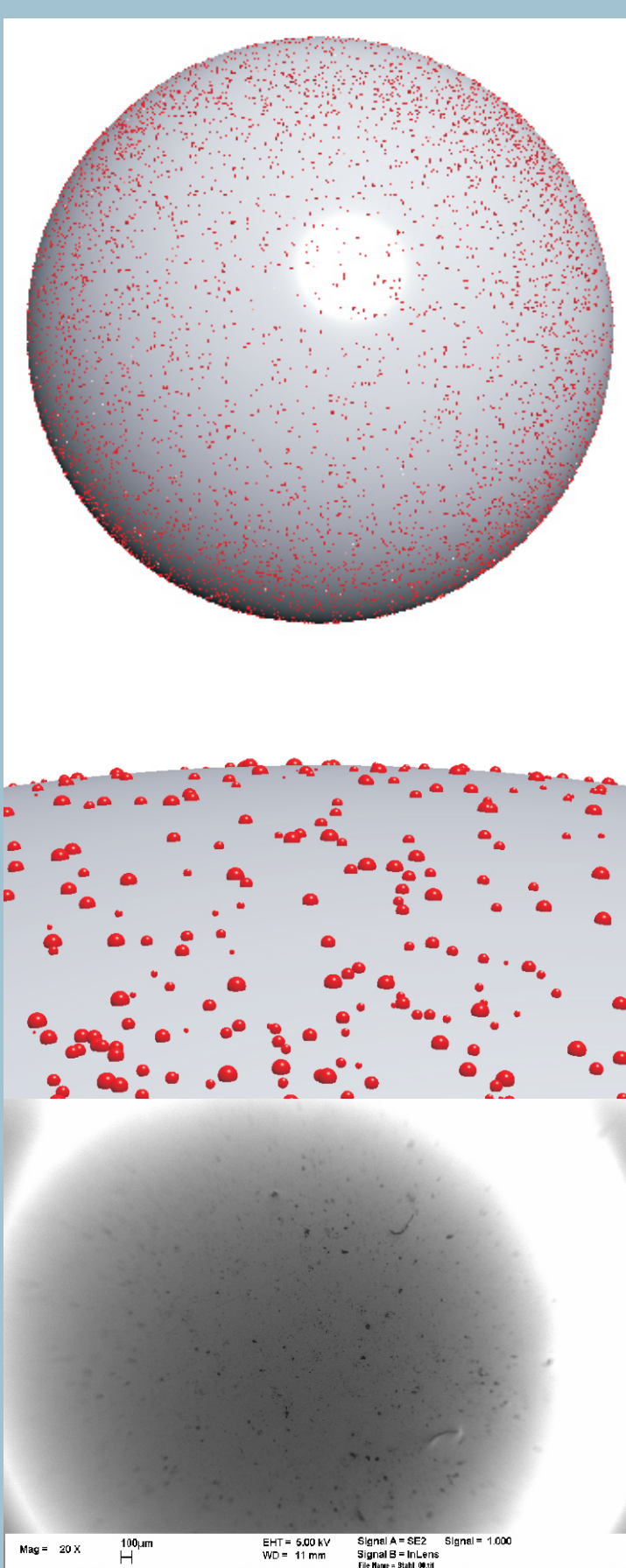
The used robot for performing large scale experiments.

Model for distribution

Assuming that the large data noise in the measurements is caused by the surface roughness of the particle, our model consists of a large central sphere of radius R and many (up to 3 million) small spheres ($R_{asp} \leq 4 \cdot 10^{-4} R$) which represent the asperities. A close-up of the simulated sphere and the microscopic image of a real steel sphere shows the similarities.

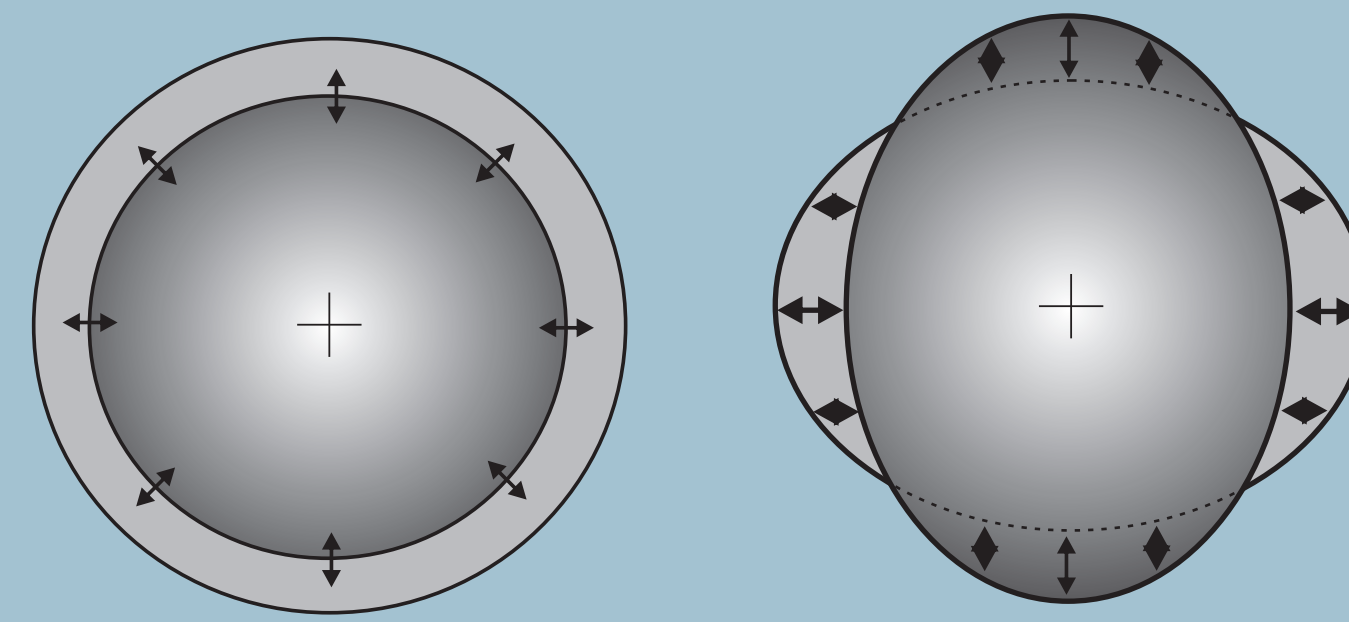


Sketch of the sphere with one asperity and the different coordinate system.



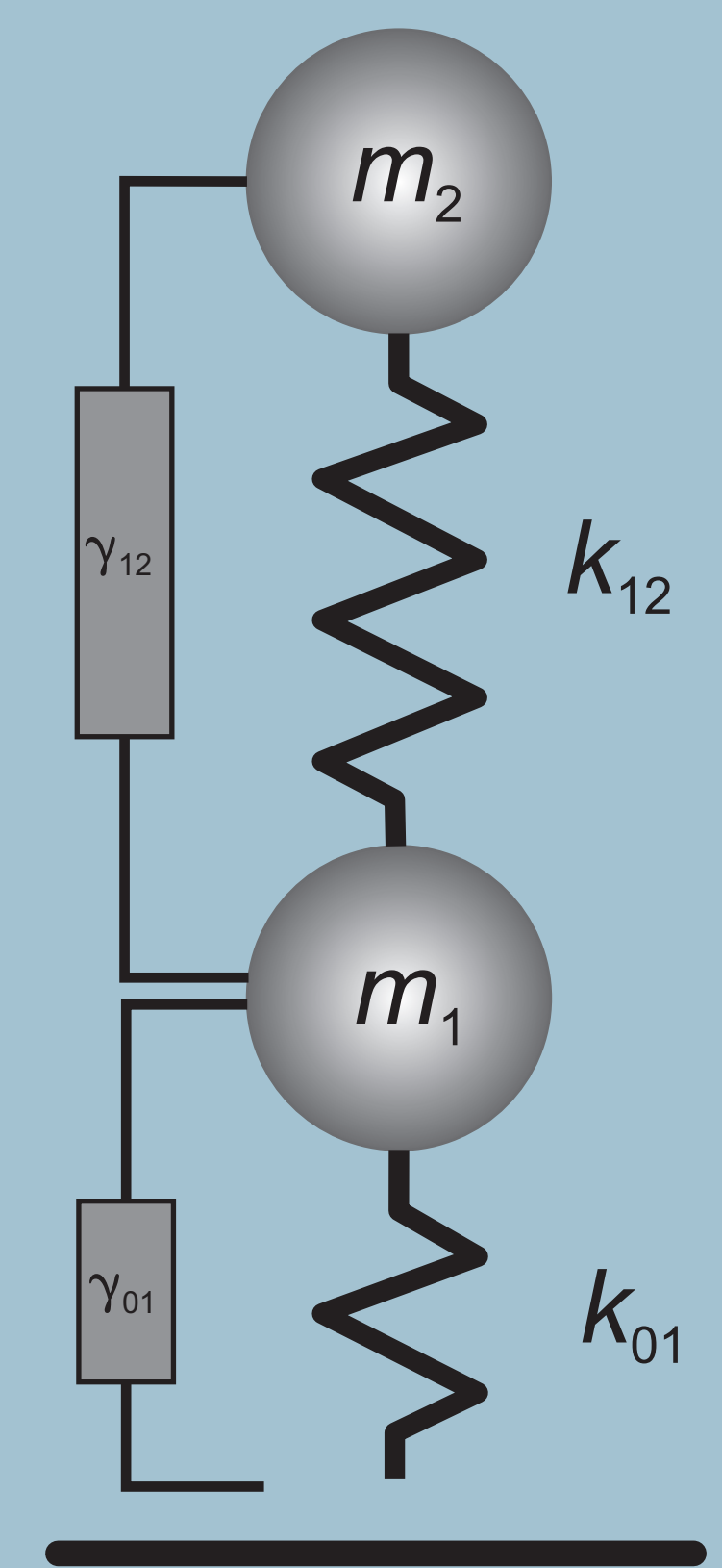
Model for steps

The steps in the measurement can be explained by stored energy in oscillatory degrees of freedom. Depending on the phase of oscillation when the sphere is contacting the surface, the coefficient of normal restitution can be increased or decreased.



Sketch of the two main eigen modes of an oscillating sphere. For a 5 mm steel sphere as used in the experiment the corresponding eigen frequencies are:

- prolate/oblate mode: $T_{prol,obl} = 3.1 \mu s$ (left)
- breathing mode: $T_{breath} = 1.8 \mu s$ (right)

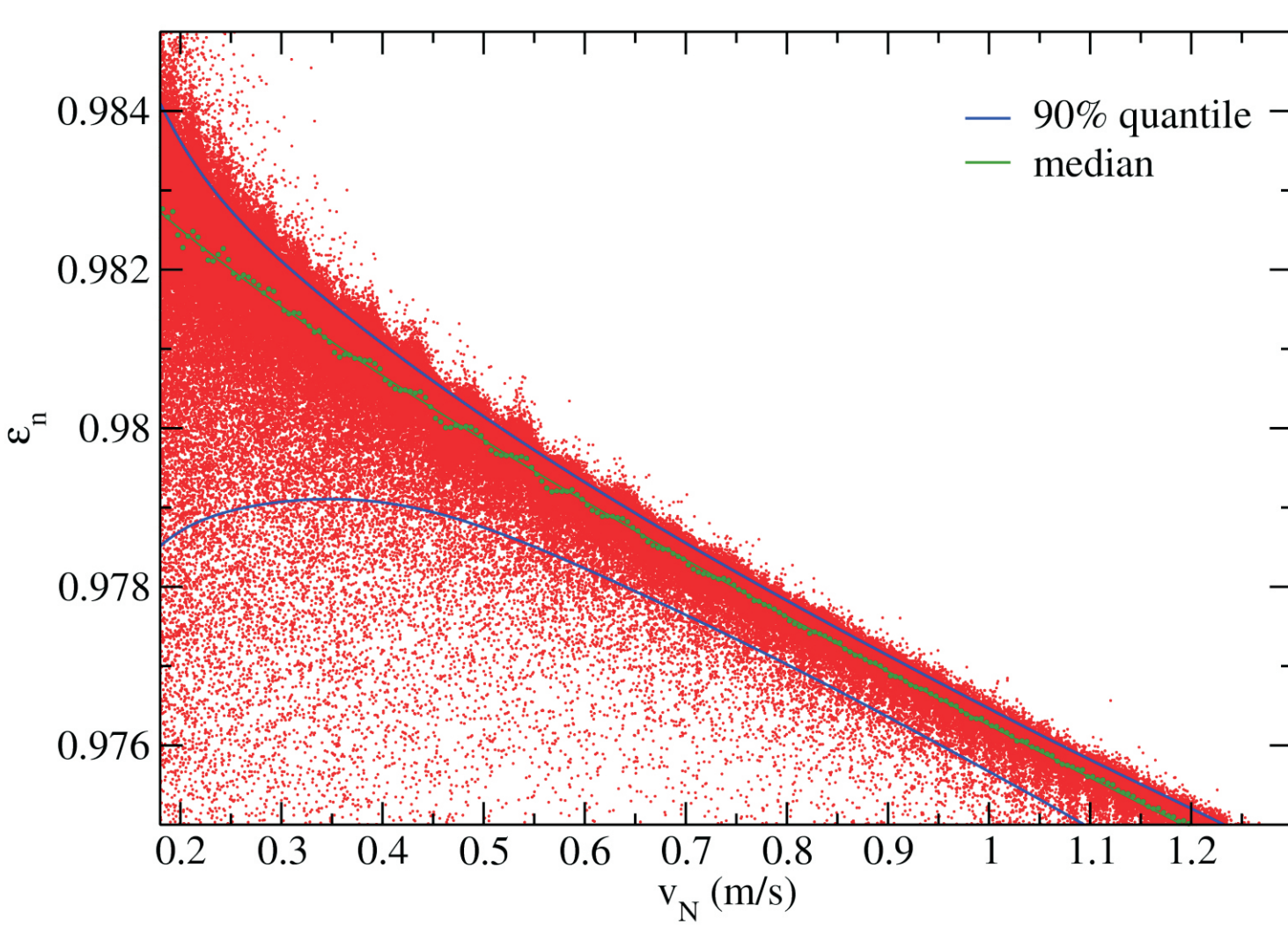
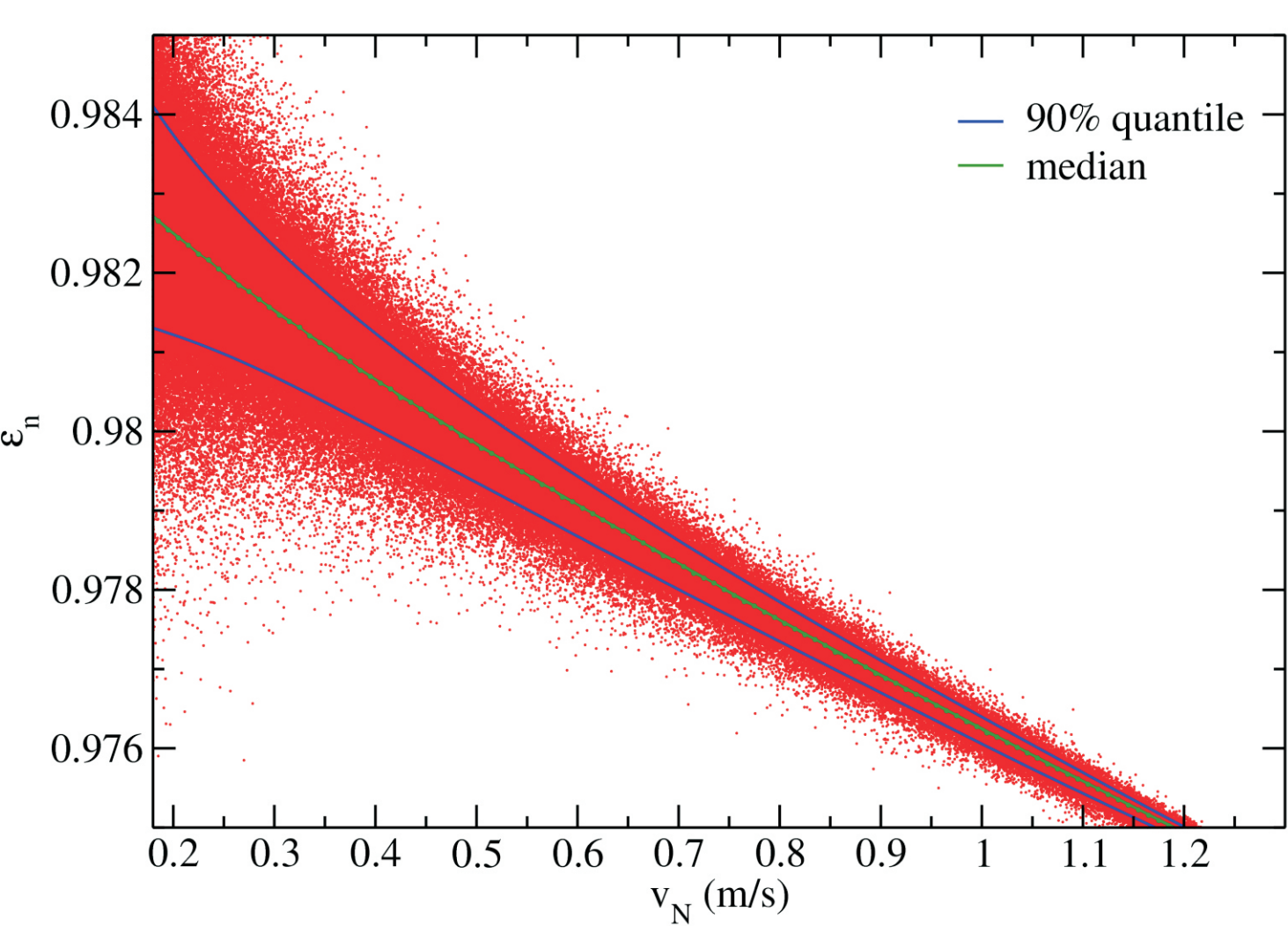


Simplified model for a body with vibrational degree of freedom.

Results for distribution

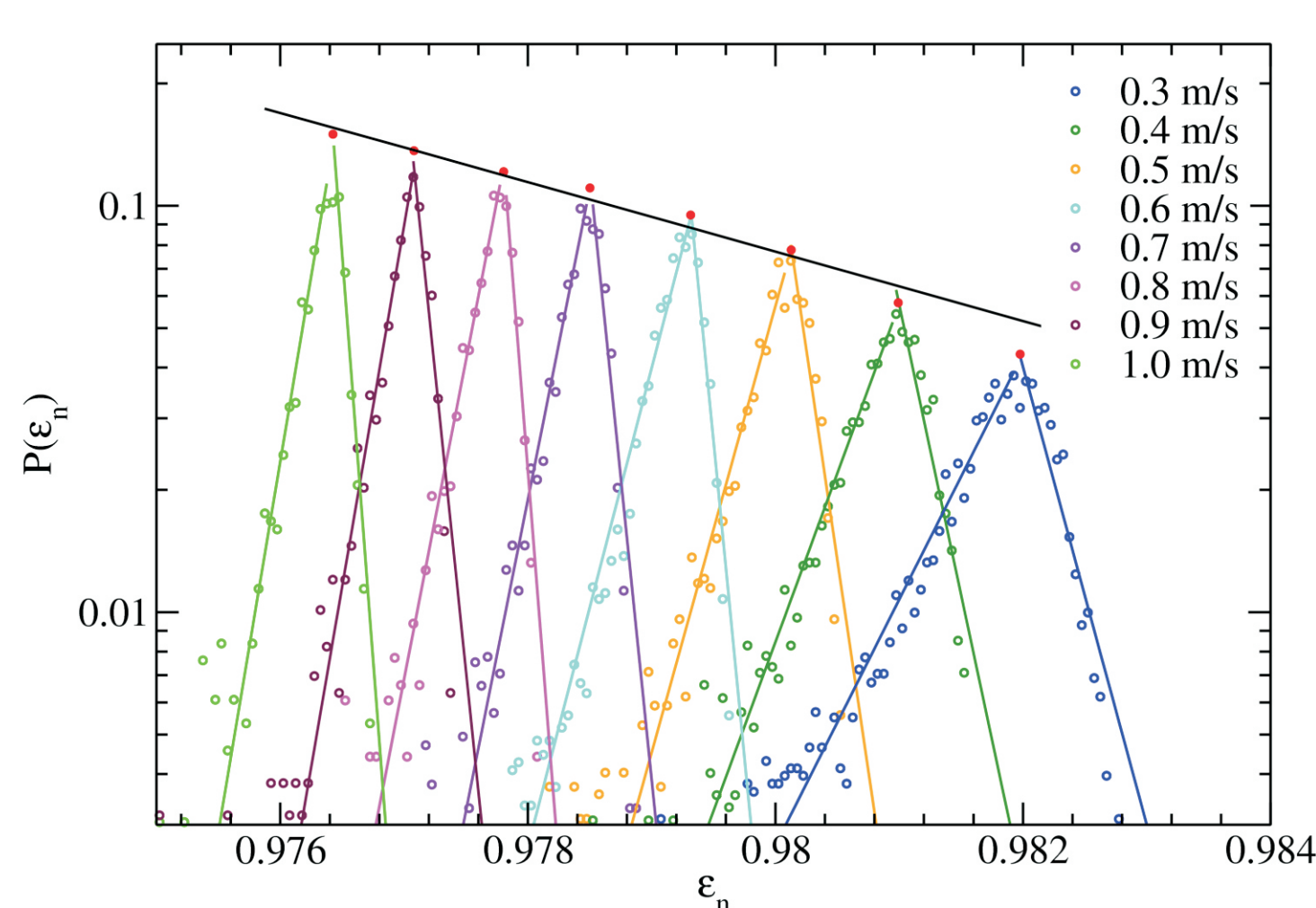
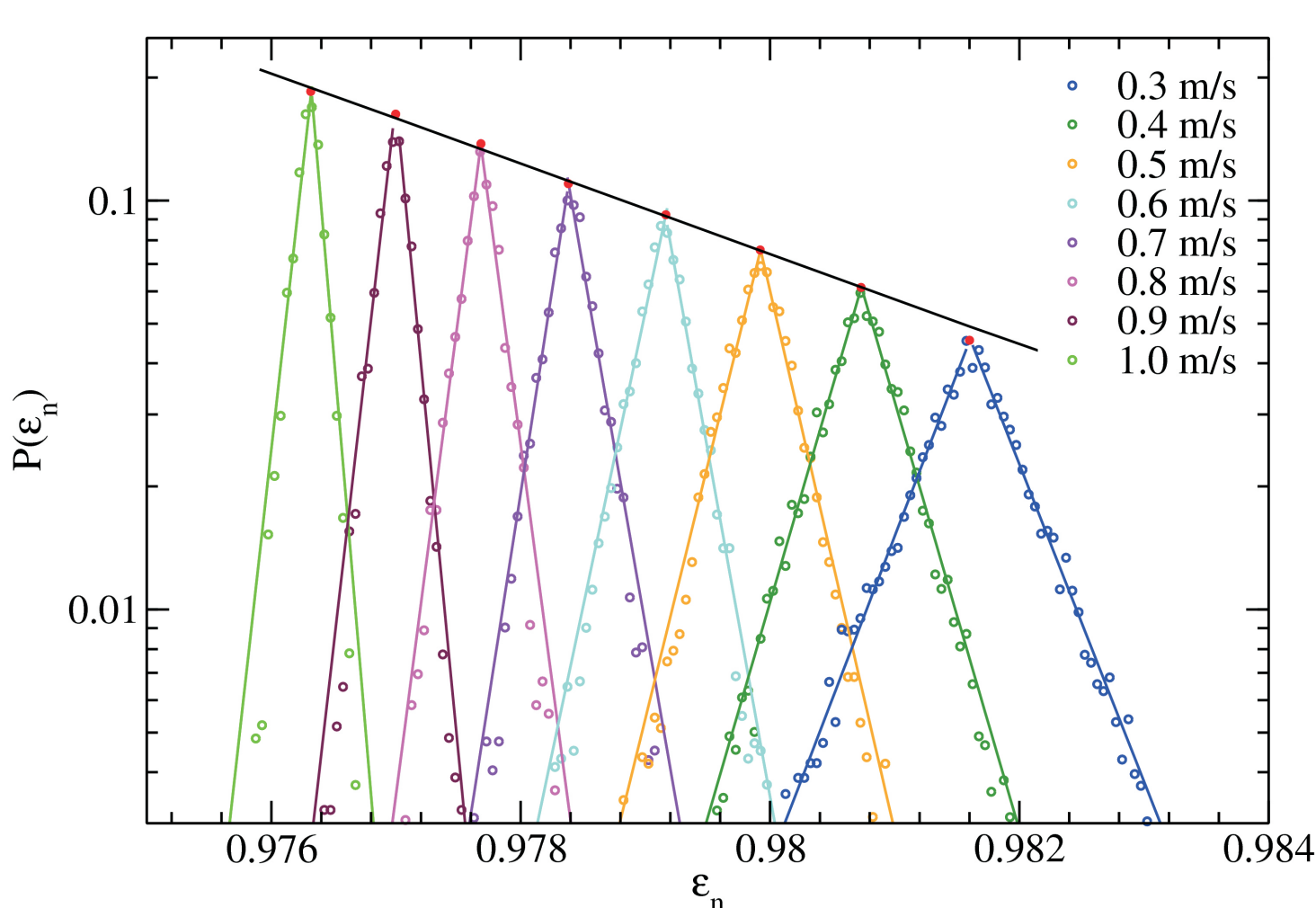
Simulation

Experiment



The coefficient of restitution ϵ plotted versus impact velocity. Between the two blue lines lie 80 % of the data points (Simulation: ~ 300.000 data points; Experiment: ~ 300.000 data points). The green line marks the median.

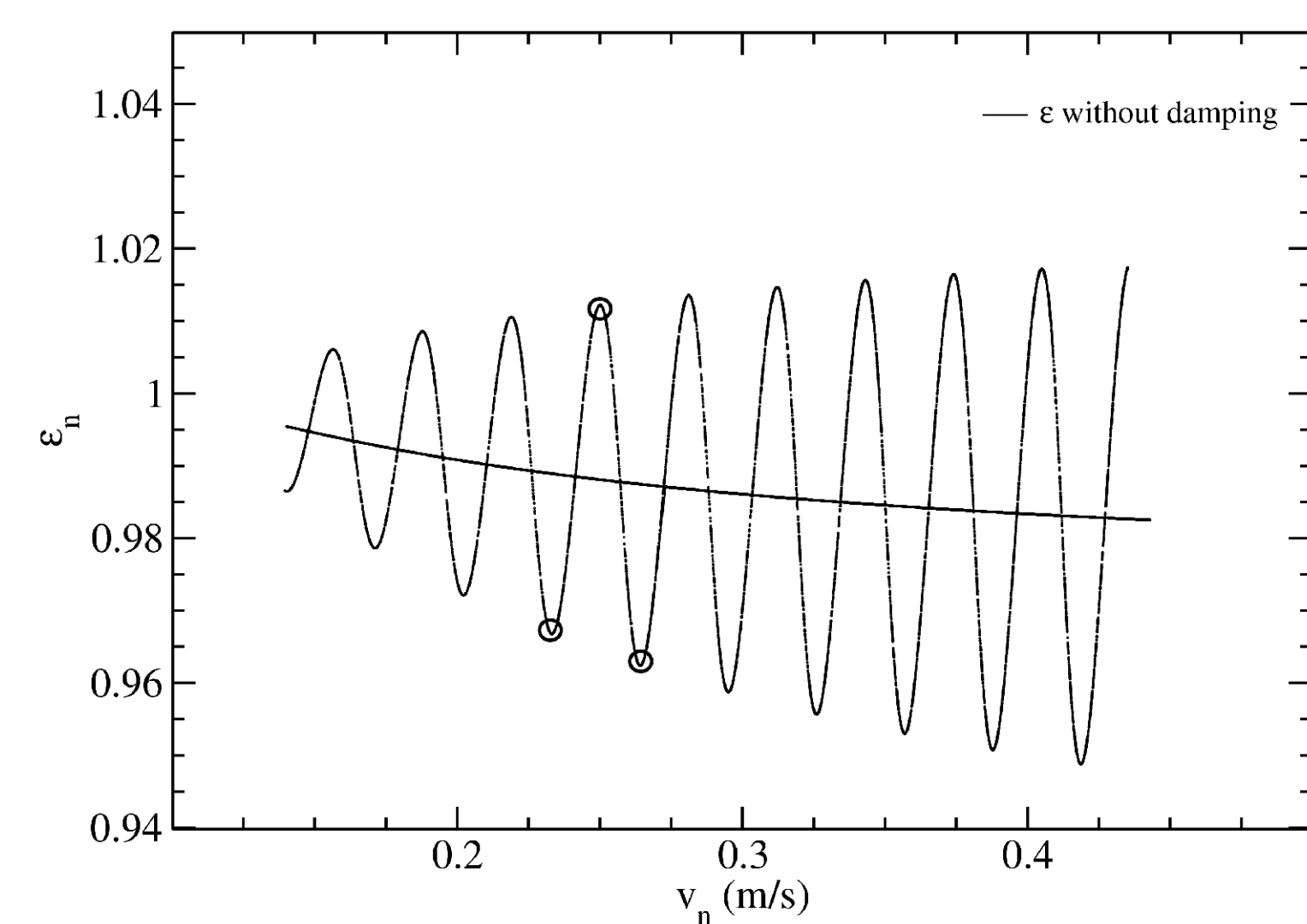
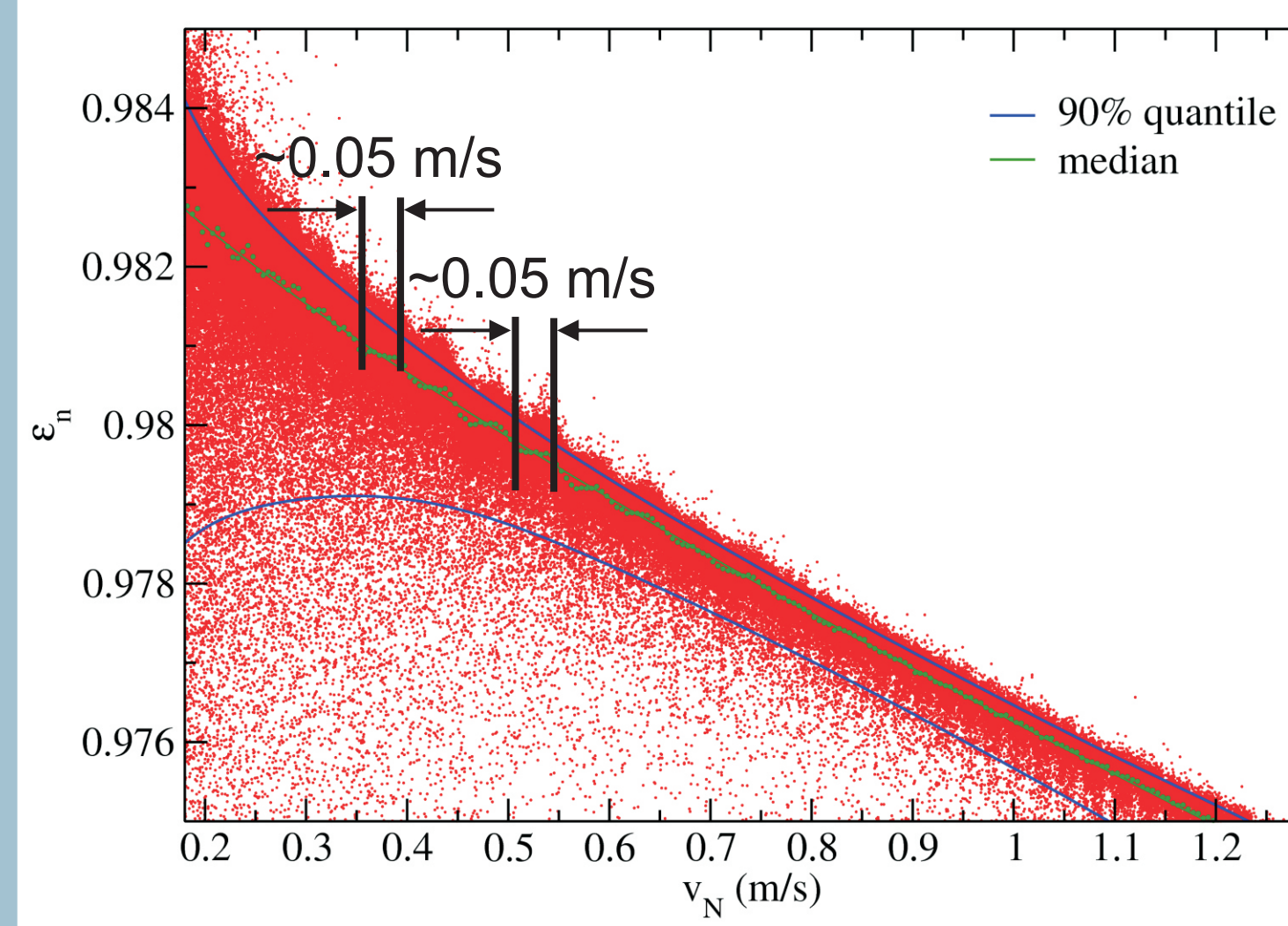
The figures below show the distribution of ϵ for different velocity intervals (0.3 m/s to 1.0 m/s; half-logarithmic scale).



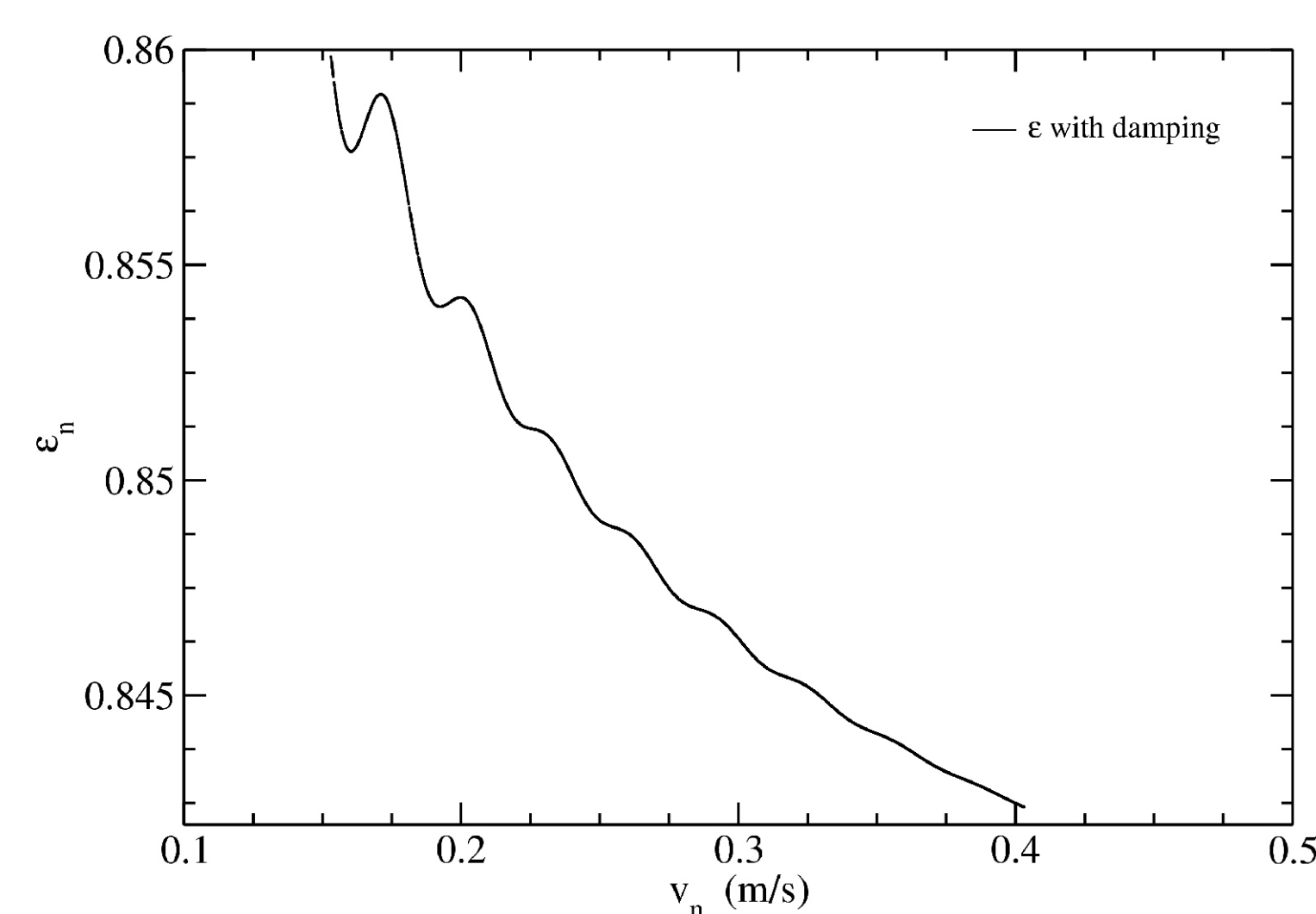
Results for steps

Experiment

Simulation



In the experimental data one can easily spot a step-like structure. The width of the steps is about 0.05 m/s and the steps are equidistant. The simple model shown above can produce a oscillation of ϵ (see figure on the upper right). The minima and maxima can be explained by the phase when the sphere is contacting the surface. With damping turned on, the curve transforms to a more step-like structure comparable with the experiment (see right figure).



Conclusion

By using the 3d model, numerical simulations show almost the same behavior as the experiment. Therefore, the microscopic asperities are able to mimic the sphere's surface roughness. We believe that the fluctuations in the measurement can be attributed to microscopic surface impurities of the sphere, which cause a transfer of translational energy into rotational energy and vice versa. We also showed that the steps we recognized in the measurement data can be explained by energy stored in vibrational modes of the setup.

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References

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