



Motion of a jumping ball due to microscopic surface impurities

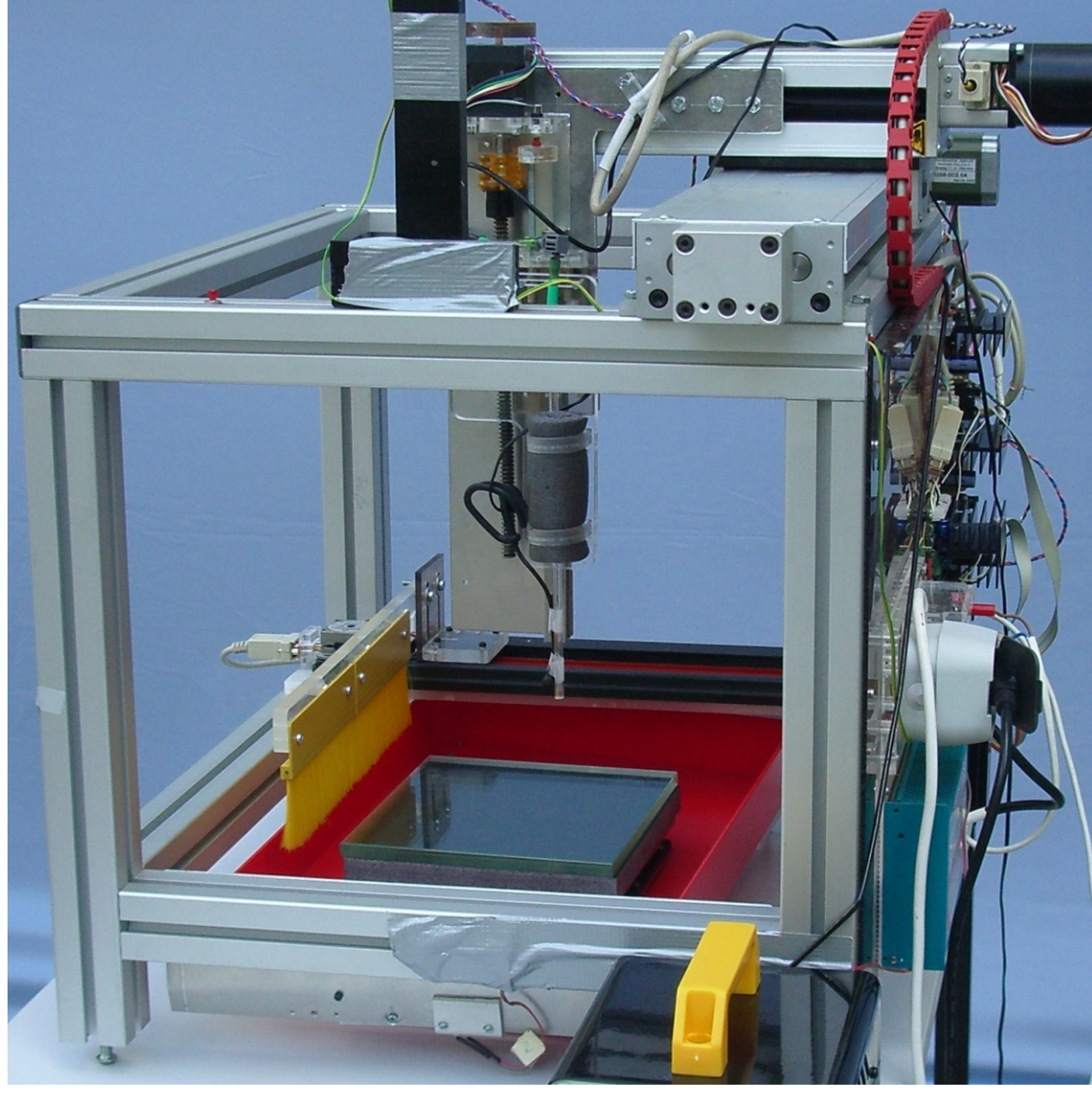


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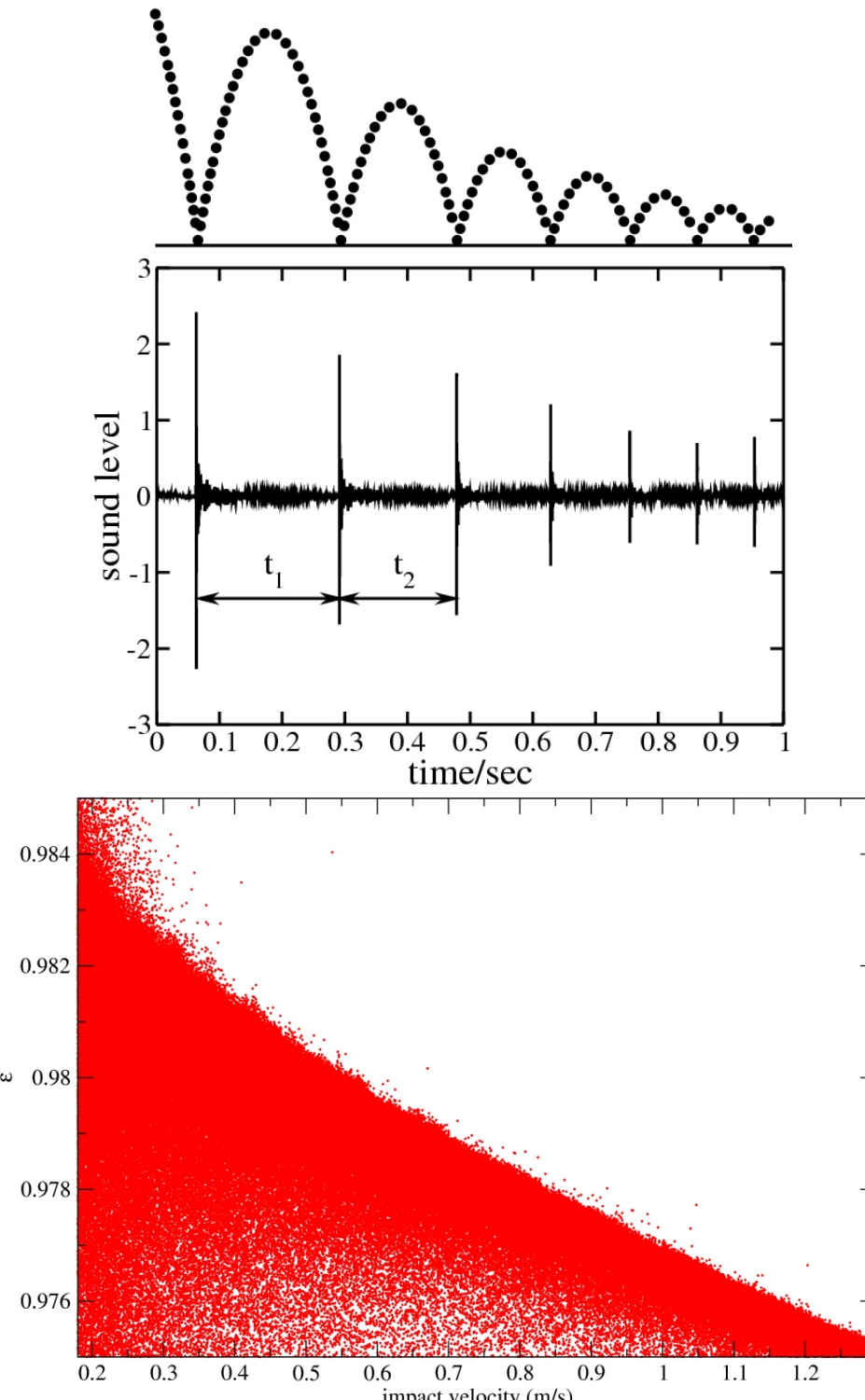
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Setup



The used robot for performing large scale experiments.

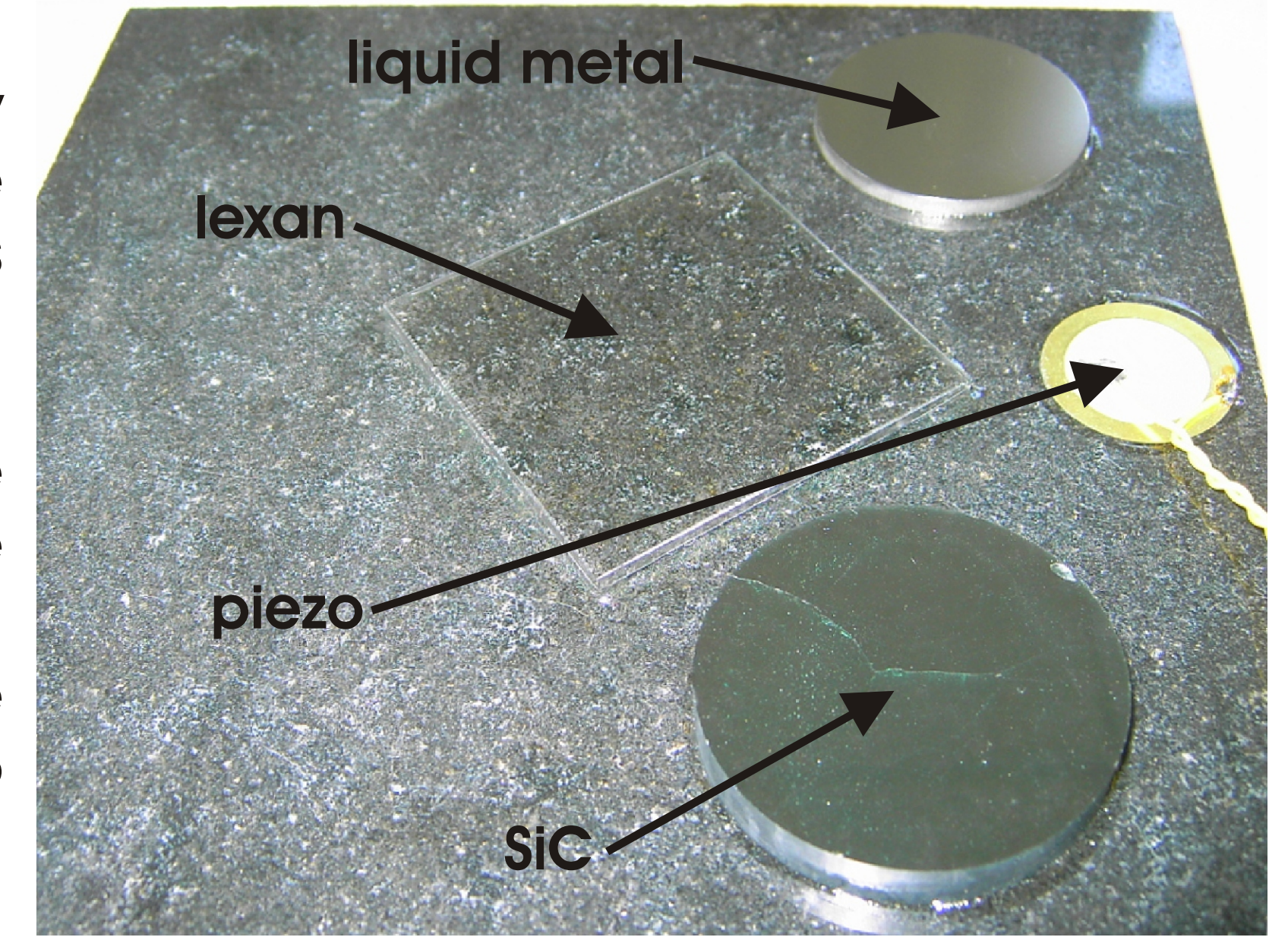


We are measuring the motion of a jumping ball using a robot which records the sound caused by a steel sphere hitting a plate. The degree of energy loss, i.e. the dissipative properties, are characterized by the coefficient of restitution. It is defined as the ratio between the post-collisional and the pre-collisional velocities.

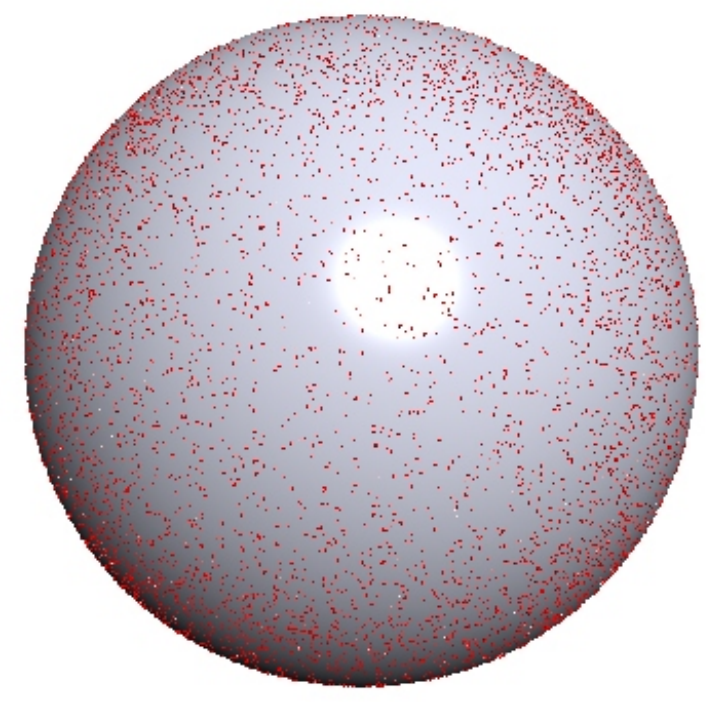
Three impacts are needed to calculate the coefficient of normal restitution $\varepsilon(v_i) = v_{2i}/v_{1i} = t_{2i}/t_{1i}$. We are using several plate materials (glass, liquid metal, SiC and lexan) to measure their influence on the coefficient of restitution. We are also analyzing the distribution of ε in the measurements.

* provided by Matthias Schröter

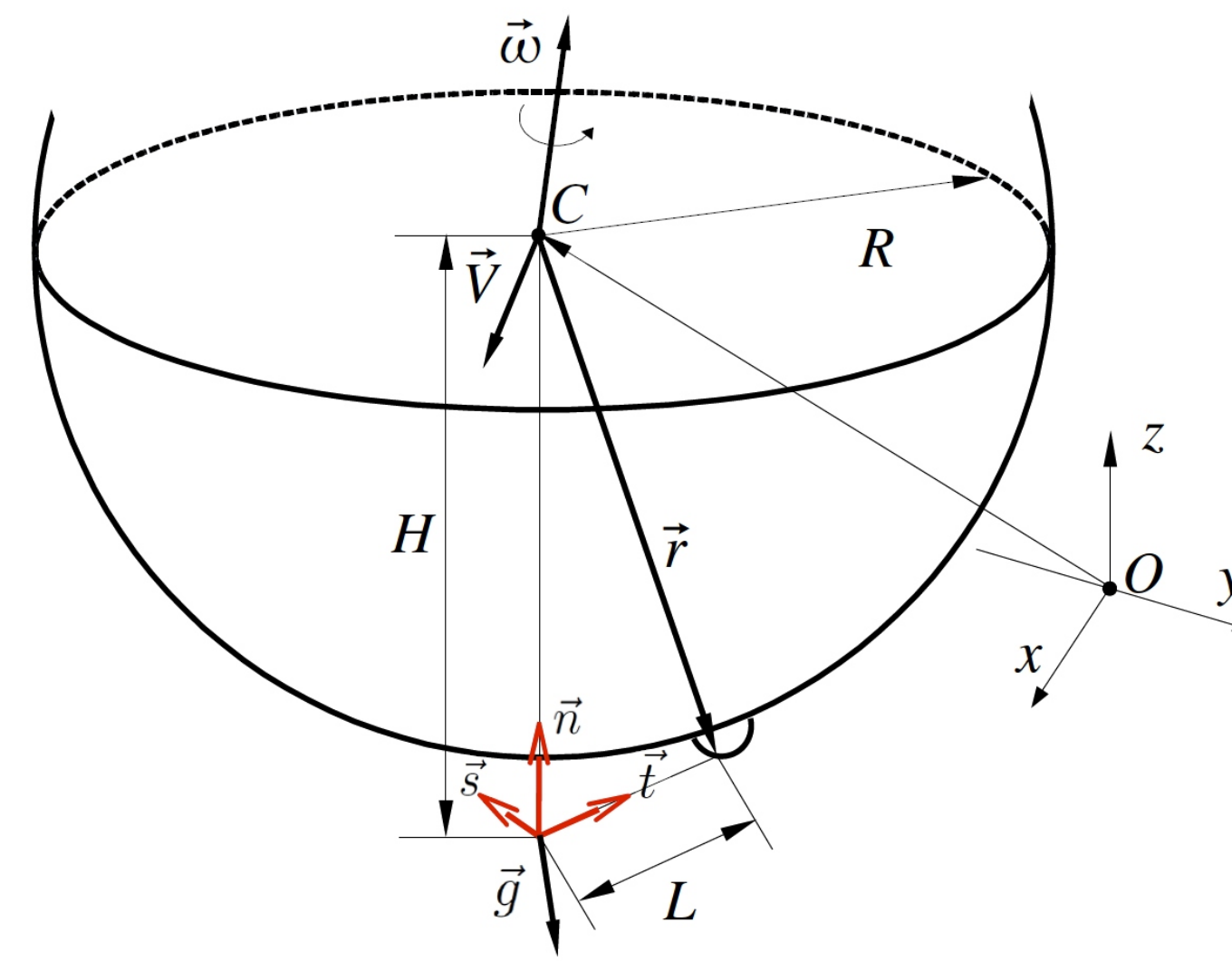
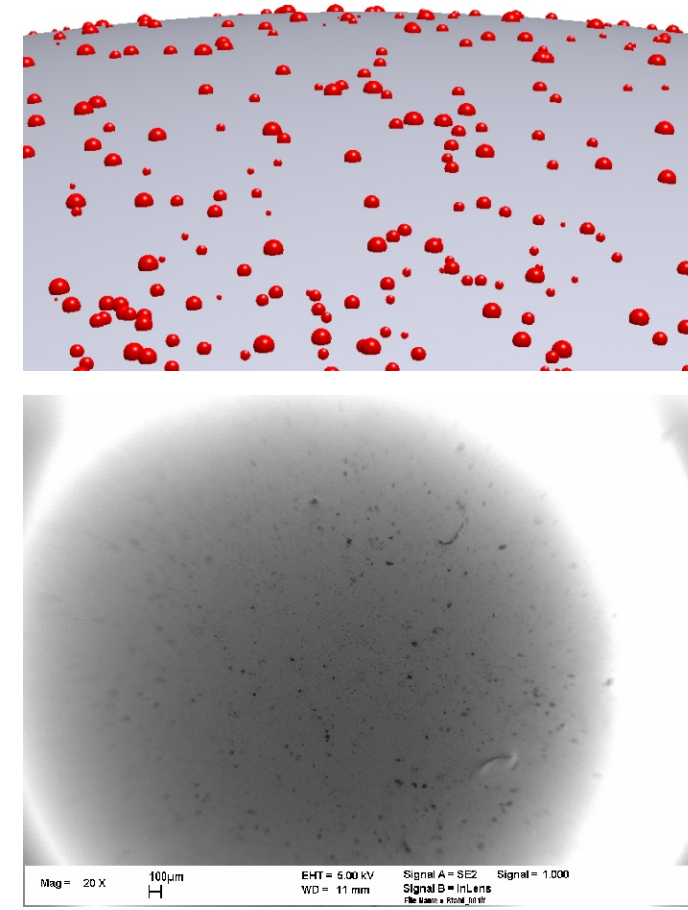
* provided by Peter Wellmann



Theoretical background



Assuming that the large data noise in the measurements is caused by the surface roughness of the particle, our model consists of a large central sphere of radius R and many (up to 3 million) small spheres ($R_{asp} \leq 4 \cdot 10^{-4} R$) which represent the asperities. A close-up of the simulated sphere and the microscopic image of a real steel sphere shows the similarities.



$$\vec{V}' = \vec{V} + \frac{\Delta \vec{P}}{m} \quad \vec{\omega}' = \vec{\omega} + \frac{\vec{r} \times \Delta \vec{P}}{J}$$

$$\vec{g} = \vec{V} + \vec{\omega} \times \vec{r} \quad \vec{g} = g_n \vec{n} + g_t \vec{t} + g_s \vec{s}$$

$$\Delta \vec{P} = P_n \vec{n} + P_t \vec{t} + P_s \vec{s}$$

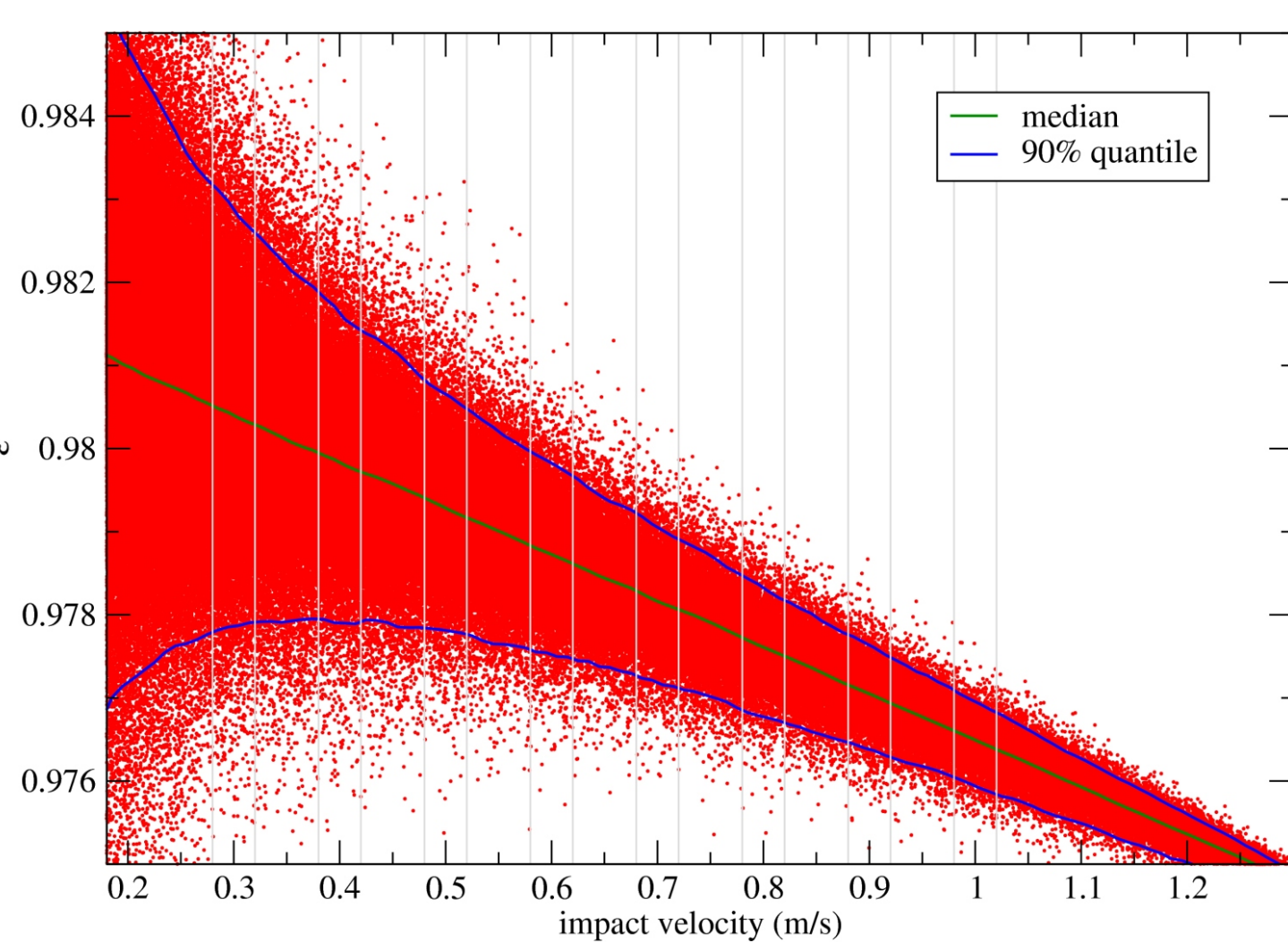
$$P_n = \frac{-m((J + mH^2)(1 + \varepsilon_n)g_n - mHL(1 - \varepsilon_t)g_t)}{J + m(H^2 + L^2)}$$

$$P_t = \frac{m(mHL(1 + \varepsilon_n)g_n - (J + mL^2)(1 - \varepsilon_t)g_t)}{J + m(H^2 + L^2)}$$

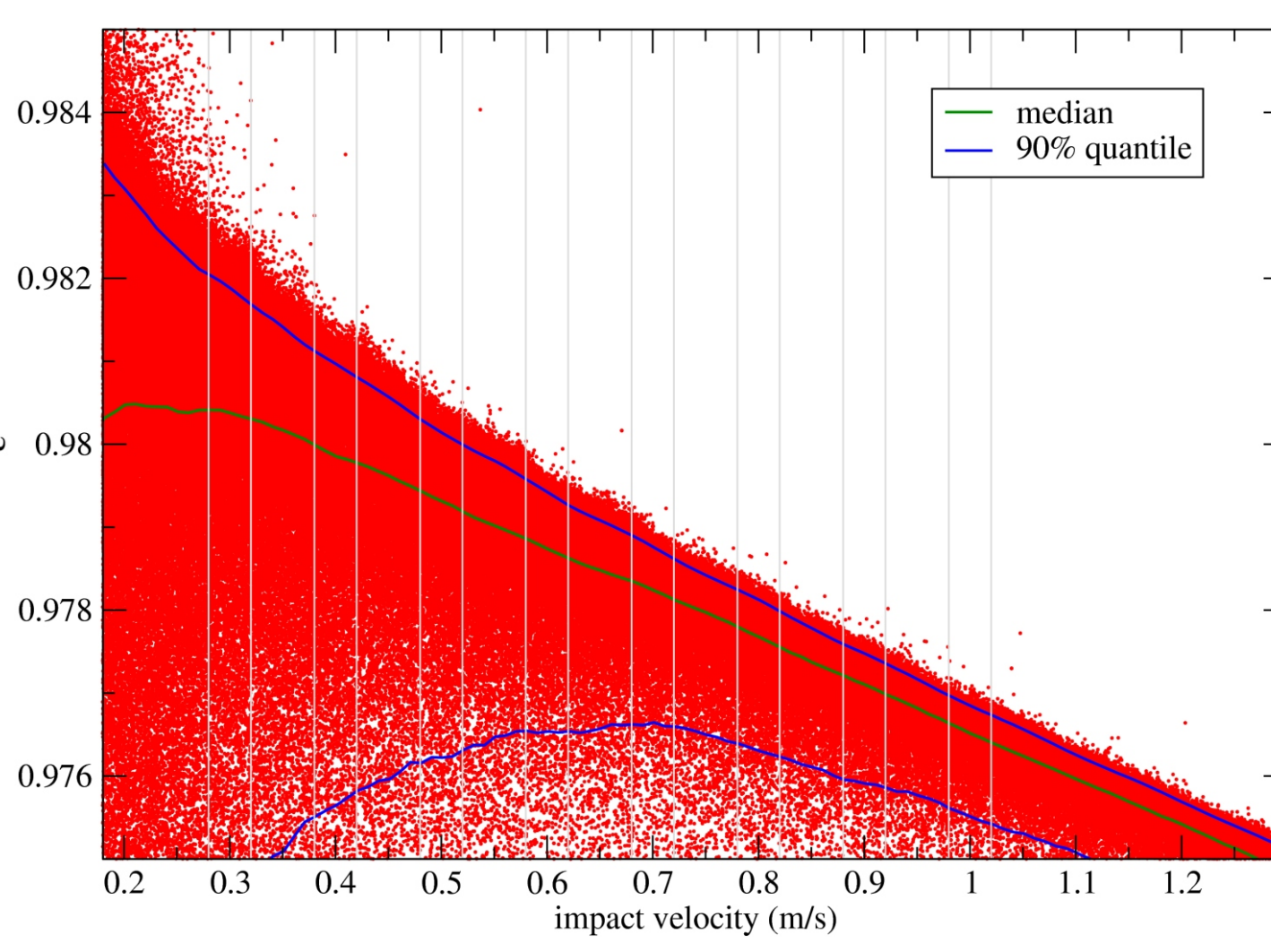
$$P_s = \frac{-mJ(1 + \varepsilon_t)g_s}{J + m(H^2 + L^2)}$$

Results

Simulation

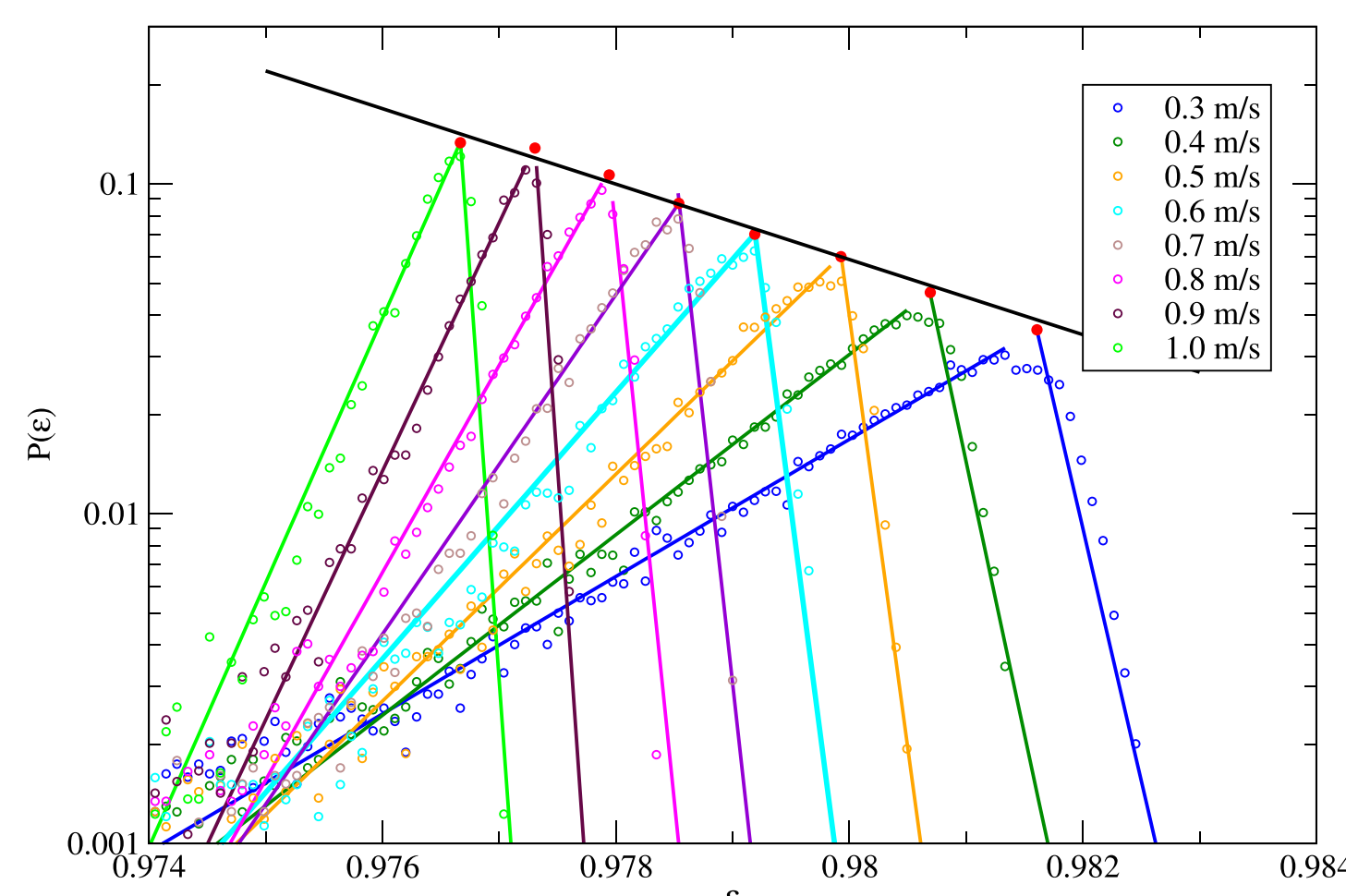
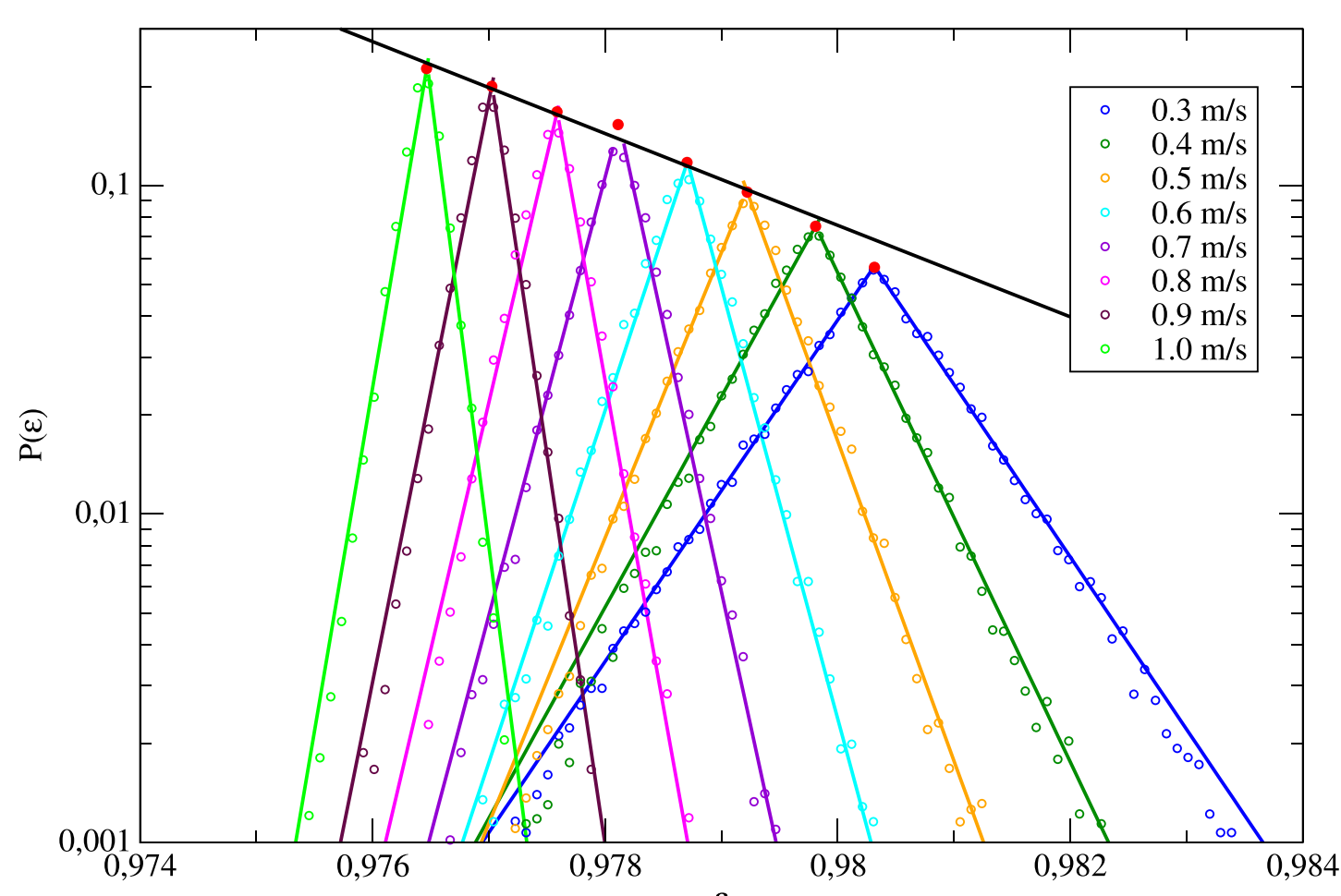


Experiment

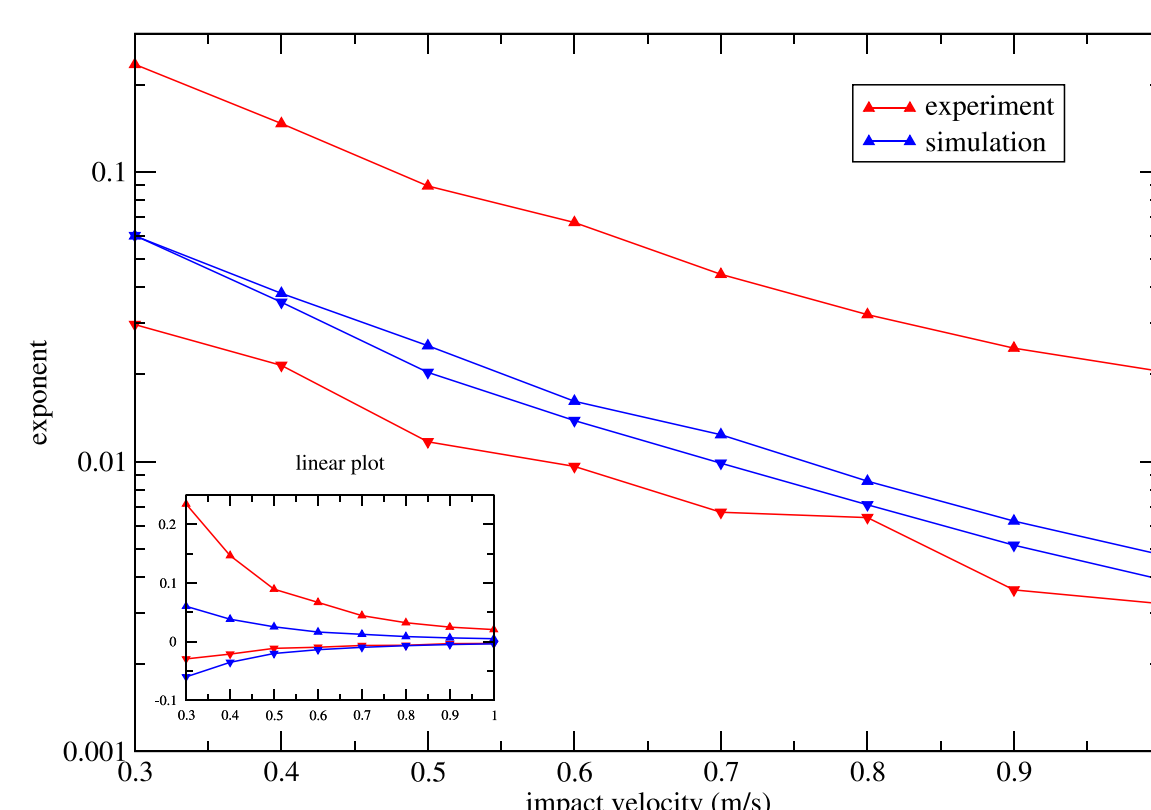


The coefficient of restitution ε plotted versus impact velocity. Between the two blue lines lie 80 % of the data points (Simulation: ~ 500.000 data points; Experiment: ~ 400.000 data points). The green line marks the median.

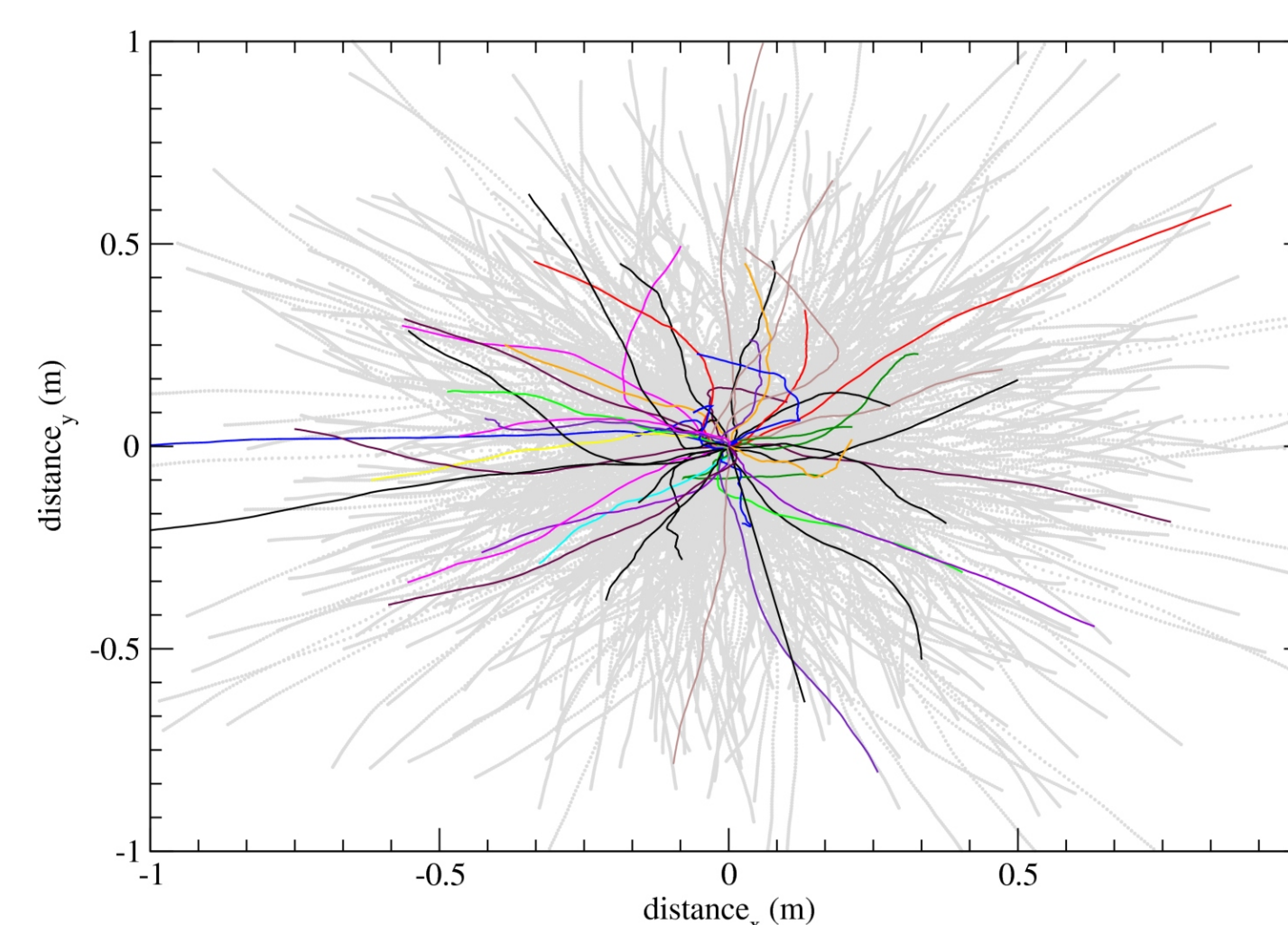
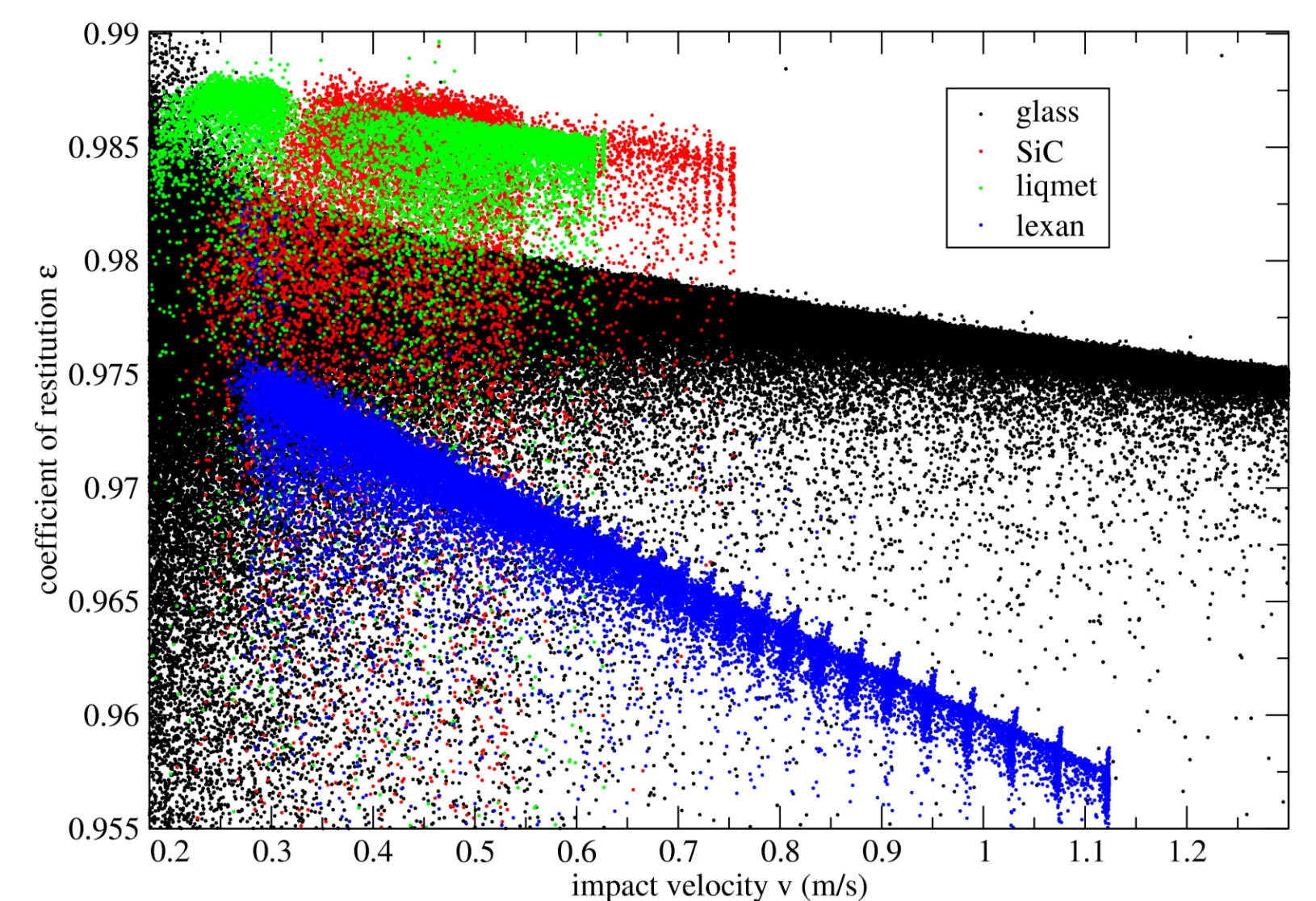
The figures below show the distribution of ε for different velocity intervalls (0.3 m/s to 1.0 m/s) marked by the grey lines in the upper plots (half-logarithmic scale).



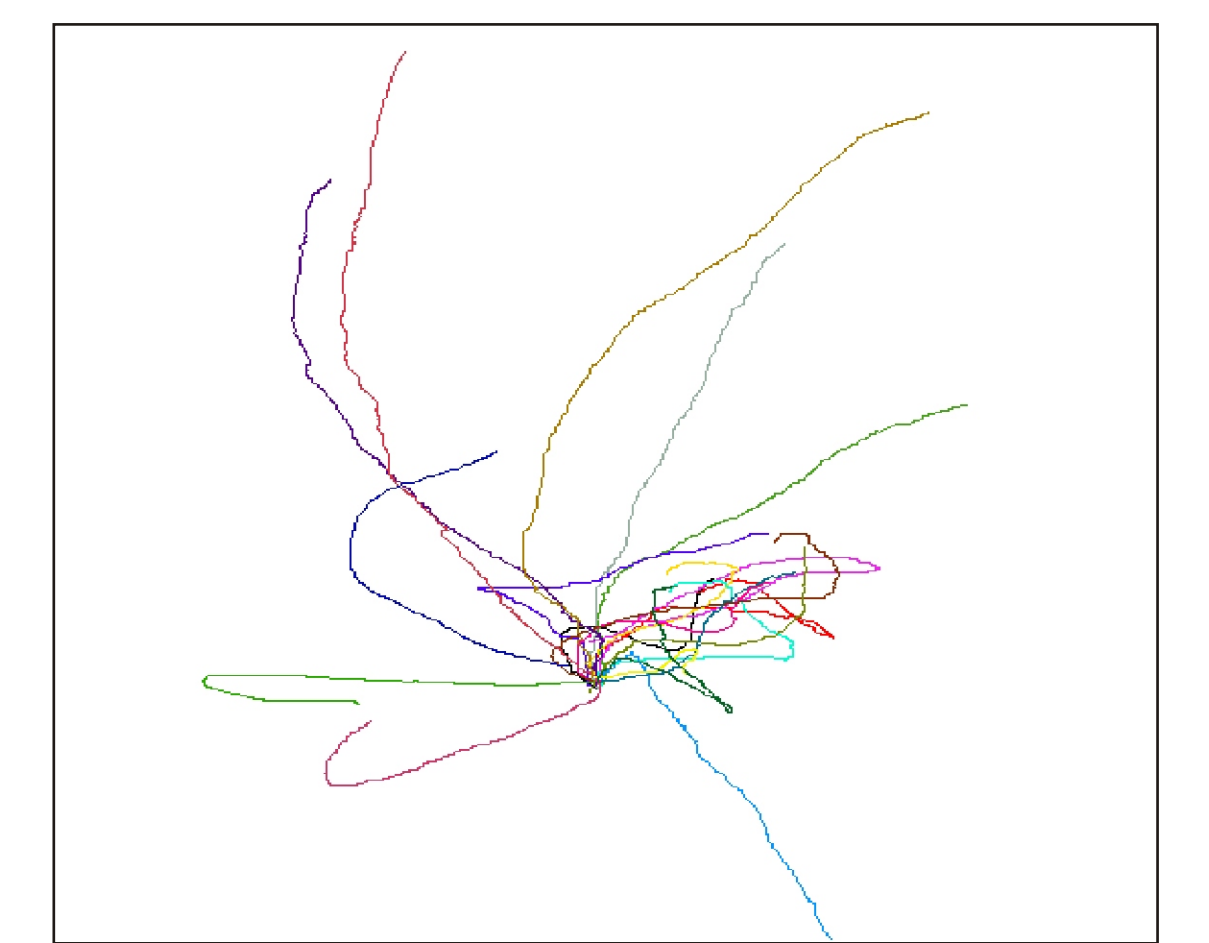
slopes



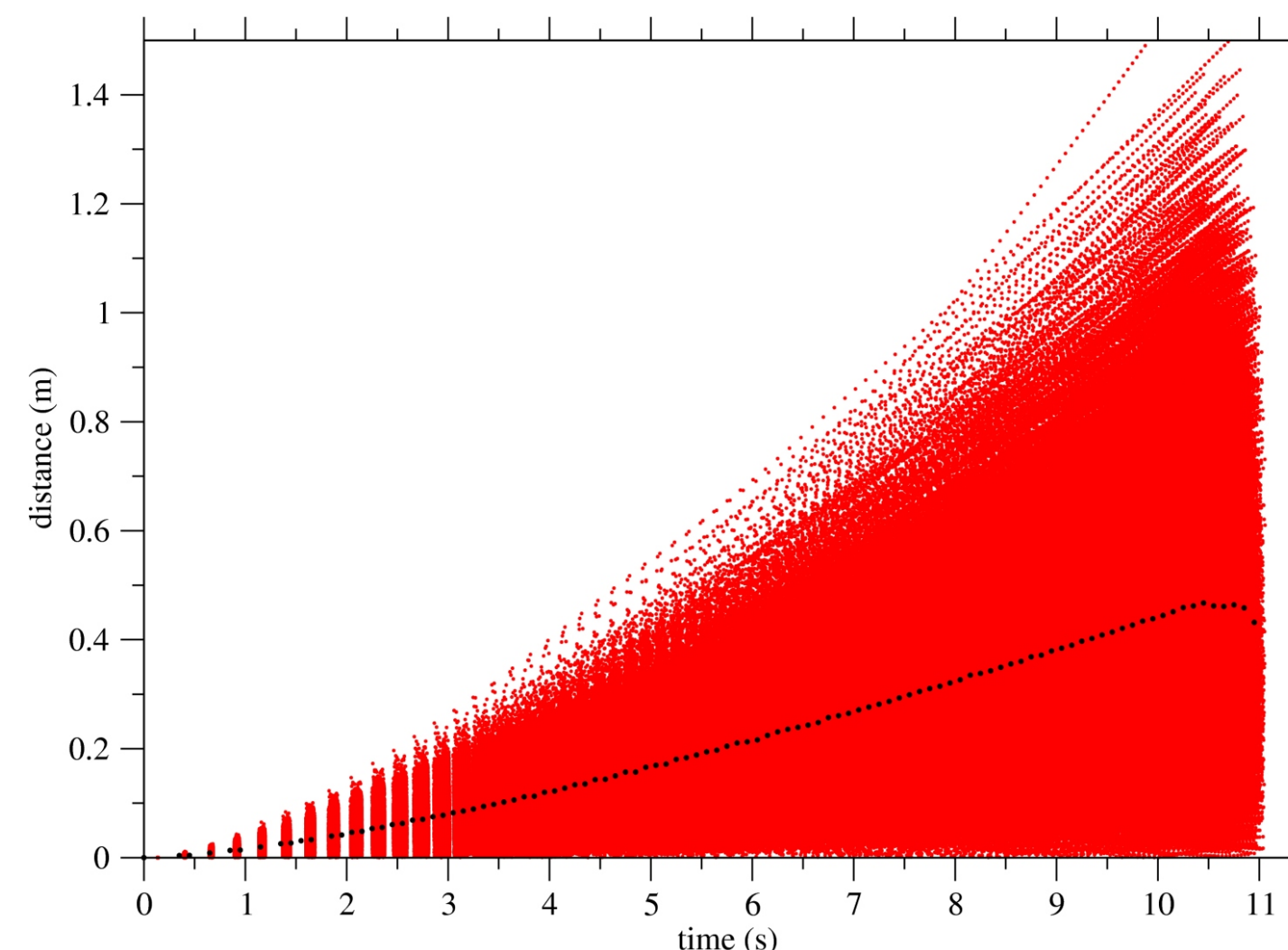
The coefficient of restitution as a function of impact velocity for four different materials.



Traces of the hopping sphere. 50 out of 100.000 simulations are colored.



Some traces from the experiment.



The displacement of the sphere over time. The circles are the averaged values over all simulations. It is not described by a random walk.

References

- T. Pöschel et. al. (2008), *EJECE* 12, 827
- I. Stensgaard and E. Lægsgaard (2001), *Am. J. Phys.* 69, 301
- A. D. Bernstein (1977), *Am. J. Phys.* 45, 41

Conclusion

By using the 3d model, numerical simulations show almost the same behaviour as the experiment. Therefore, the microscopic asperities are able to mimic the sphere's surface roughness. We believe that the fluctuations in the measurement can be attributed to microscopic surface impurities of the sphere, which cause a transfer of translational energy into rotational energy and vice versa.

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