Non-Uniformities in the Angle of Repose and Packing Fraction of Large Heaps of Granular Matter

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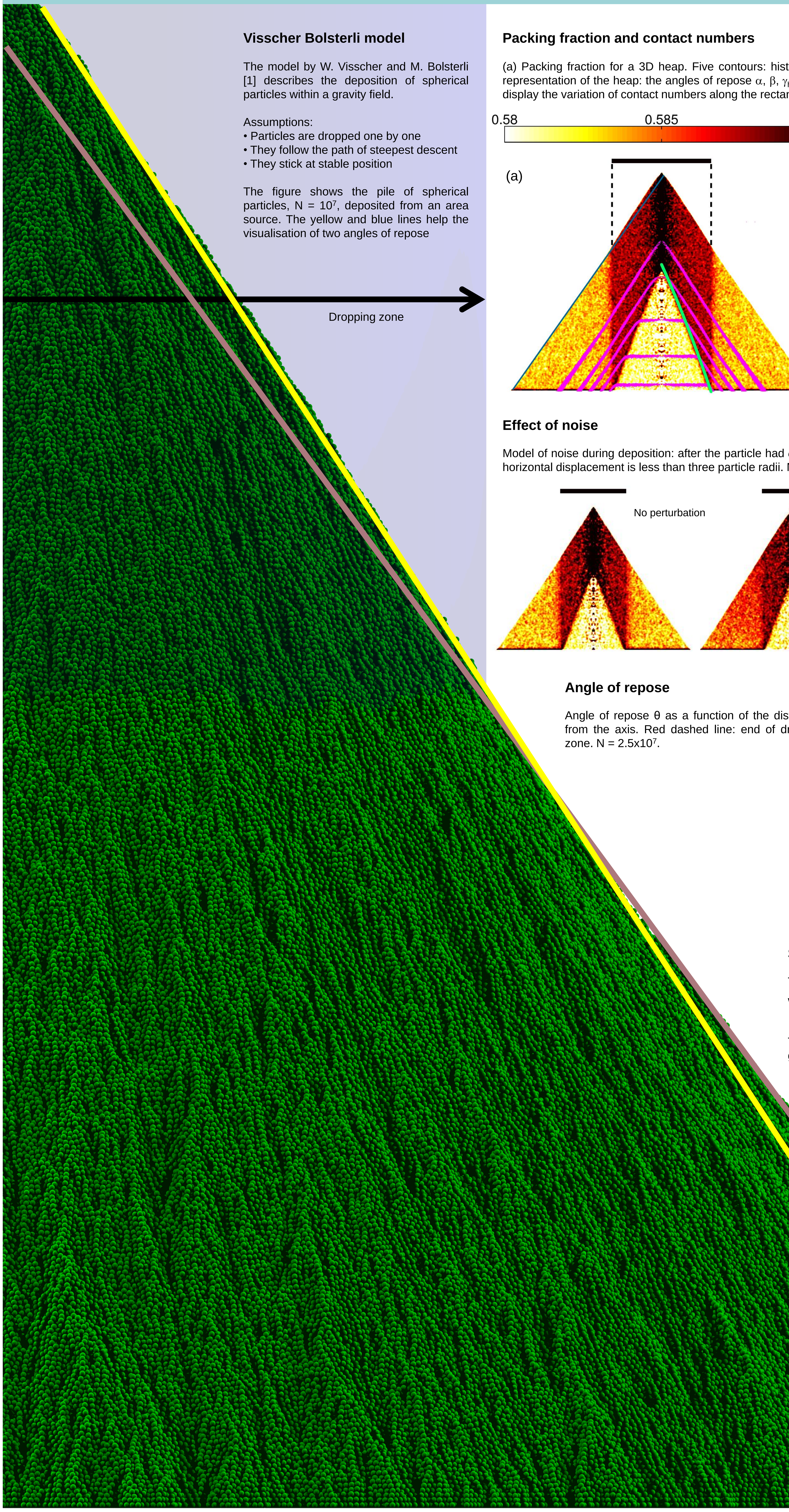
We report a numerical investigation of the structural properties of very large three-dimensional heaps of granular material produced by ballistic deposition from extended circular dropping areas. Very large heaps are found to contain three new geometrical characteristics not observed before: they may have two external angles of repose, an internal angle of repose, and four distinct packing fraction (density) regions. Such characteristics are shown to be directly correlated with the size of the dropping zone. In addition, we also describe how noise during the deposition affects the final heap structure.



Visscher Bolsterli model

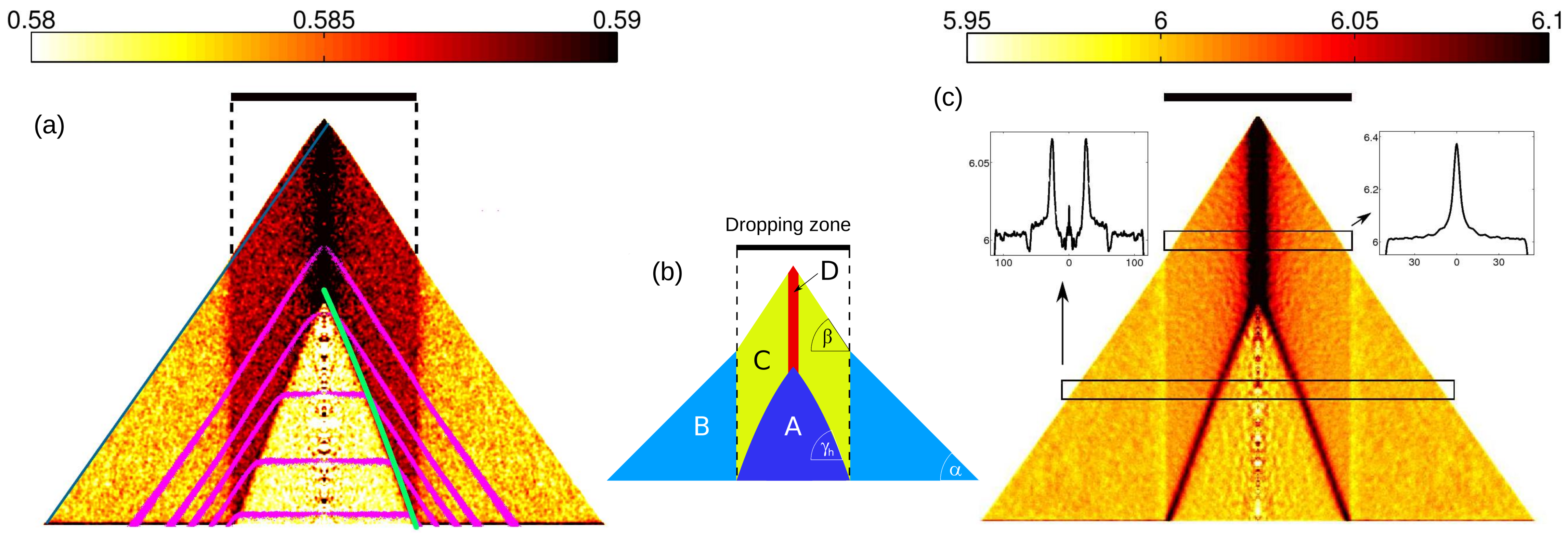
The model by W. Visscher and M. Bolsterli [1] describes the deposition of spherical particles within a gravity field.

Assumptions:
- Particles are dropped one by one
- They follow the path of steepest descent
- They stick at stable position

The figure shows the pile of spherical particles, $N = 10^7$, deposited from an area source. The yellow and blue lines help the visualisation of two angles of repose

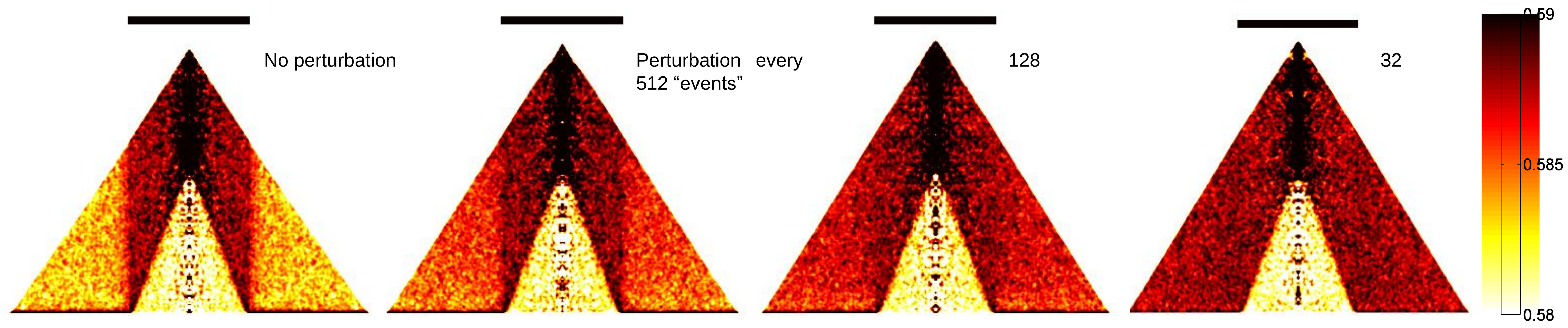
Packing fraction and contact numbers

(a) Packing fraction for a 3D heap. Five contours: history of the heap. Green line: prediction of equation (2). Number of particles is $N=10^7$. (b) Schematic representation of the heap: the angles of repose α, β, γ_h, and four density zones A, B, C and D. (c) The average contact numbers inside the heap. The insets display the variation of contact numbers along the rectangles.



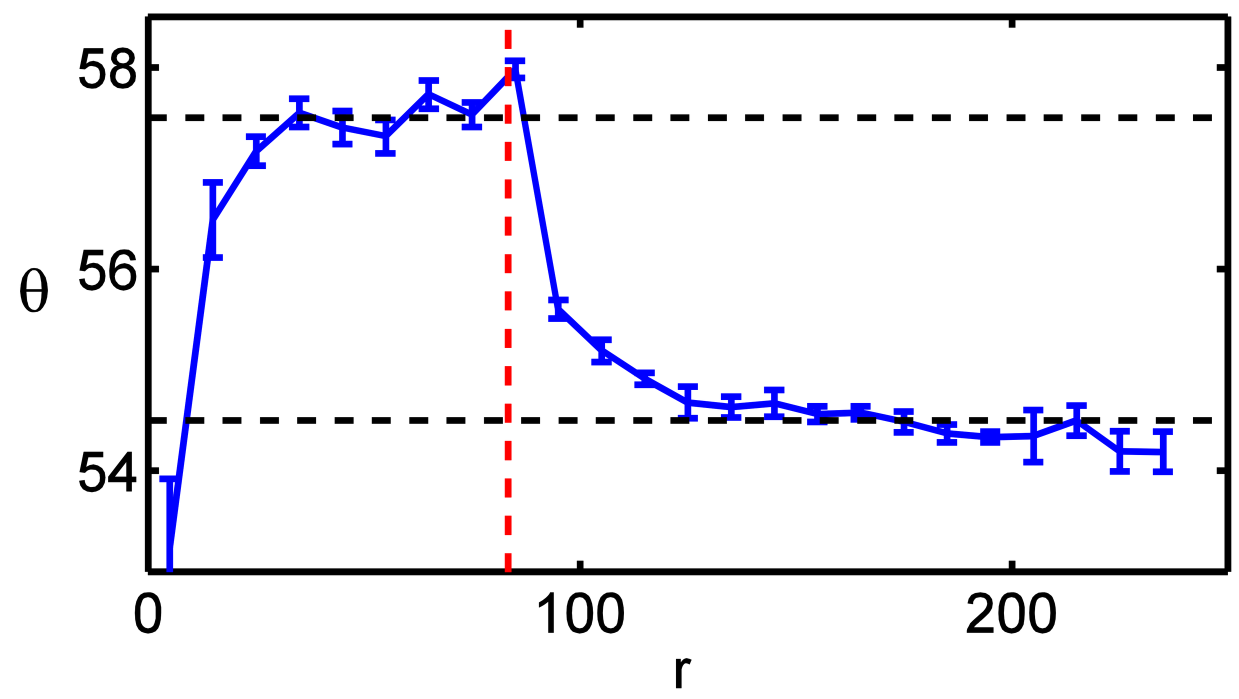
Effect of noise

Model of noise during deposition: after the particle had *e* of events in the pile we vertically lift the particle and drop it randomly at a nearby location, such that horizontal displacement is less than three particle radii. $N = 3\times10^6$.



Angle of repose

Angle of repose θ as a function of the distance *r* from the axis. Red dashed line: end of dropping zone. $N = 2.5\times10^7$.



Shape of area A

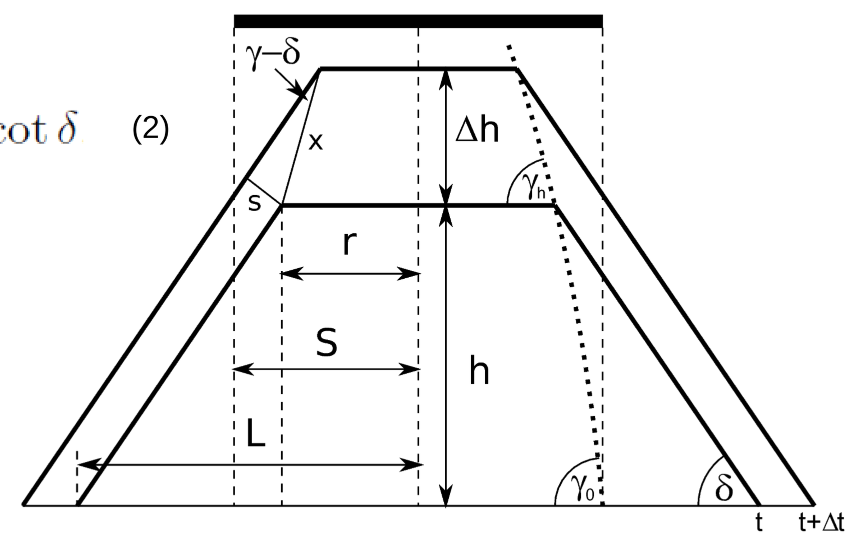
The angles α, β, and γ_h are not independent:

$$\tan\gamma = \frac{(r + h\cot\delta)^2 - r^2}{(r + h\cot\delta)^2 - S^2}\tan\delta \qquad (1)$$

We assume *δ=(α+β)/2*.

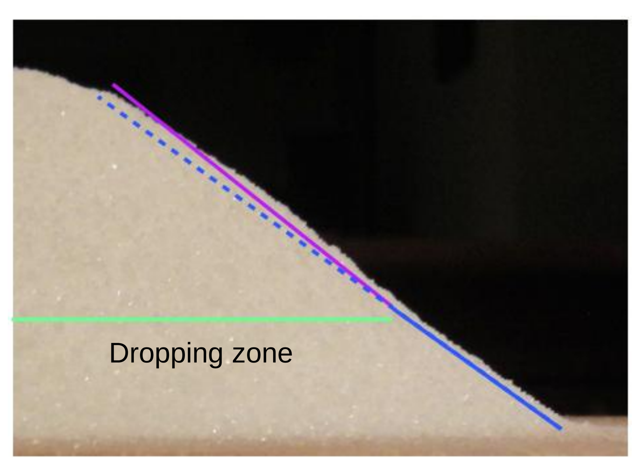
The flat surface of the heap shrinks during growth giving the shape of area A:

$$\frac{dr}{dh} = -\frac{(r + h\cot\delta)^2 - S^2}{(r + h\cot\delta)^2 - r^2}\cot\delta \qquad (2)$$



Kitchen table experiment

Pile of sugar displaying two angles of repose.



References:
[1] W. M. Visscher and M. Bolsterli, Nature **239**, 504 (1972).
[2] R. Jullien and P. Meakin, Nature **344**, 425 (1990).
[3] N. Topic, J. A. C. Gallas and T. Pöschel, submitted for publication.

In conclusion, the analysis of very large three-dimensional heaps of granular matter proves to be quite revealing. We find such heaps to be characterized by several new geometrical features: (i) two external angles of repose; (ii) an internal angle of repose, and (iii) four distinct density (packing fraction) regions. We have also found that duality of the angle of repose may be washed out by moderate to strong noise during the deposition process.