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Pattern formation in a horizontally shaken granular submonolayer

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Abstract We study the mechanism leading to the formation of stripe-like patterns in a rectangular container filled with a sub-monolayer of frictional spherical particles when it is subjected to horizontal oscillations. By means of Molecular Dynamics simulations we could reproduce the experimental results. Systematic simulations allow to identify friction to be responsible for the pattern formation, that is, the tangential interaction between contacting particles and between the particles and the floor of the container. When particles are in contact with the floor and other adjacent particles simultaneously, there emerges a frustrated situation in which the particles are prevented from rolling on the floor. This effect leads to local jamming and eventually to stripe-like pattern formation. In the long time evolution, the stripes are unstable. Stripes may merge as well as disintegrate.

Keywords Pattern formation · Horizontal shaking · Frustration effects

1 Introduction

In many situations, granular systems subjected to external agitation have a tendency to segregate, rather than to mix. This phenomenon is of enormous importance in industrial applications, see [1–3] for review. When mixtures of particles differing in size, shape or material properties are agitated, a variety of pattern formation phenomena is observed, e.g. [4–6]. The effect discussed in this paper belongs to the class of stratification phenomena, that is, as the result of a dynamical process, the particles arrange themselves in form of stripes. Stratification in granular systems is a rather general process and can be found in various systems. The most prominent example is stratification occurring when a mixture of small and large particles (or particles differing in shape) are poured from a point source to form a heap and there exists extensive literature on this effect, e.g. [7–23], which is caused by different angles of repose depending on the particle types [24,25]. In geological three-dimensional systems these stratification patterns may adopt rather complex structures [7,8]. Frequently, stratification patterns can be found in geological systems, e.g., regular stripes of stones are found on many non vegetated alpine and polar hill slopes [26]. Albeit a variety of mechanisms have been proposed [27], their spontaneous formation is still unclear.

Structure formation is also observed in vertically or horizontally periodically driven shallow granular systems. In the case of vertically vibrated granular matter the energy is injected into the system through collision with the bottom plate. In a certain range of driving parameters, separation into solid-like and fluid-like regions is observed where the phases differ significantly in density, local order and granular temperature [28–30]. Here clustering takes place due to the increase of the dissipation rate with the increase of the density of the granular gas [31].

For vertical excitation, particles are immobile for small values of the driving force. When the driving force exceeds a critical value, isolated particles begin to move. This may happen just below the amplitude of the acceleration reaches gravity [32]. If the driving force exceeds a second threshold, the granular medium forms a granular gas. Between these threshold values the gas phase is unstable and a fraction of particles can form immobile or slowly moving clusters, possibly undergoing coarsening dynamics of Ostwald-ripening type: small clusters disappear and large clusters grow with time [31]. For the case of sub-monolayers, besides structure

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formation, deviations from the Gaussian velocity distribution function were observed in experiments and simulations [30,33,34]. These deviations are most pronounced at low forcing and vanish as forcing is increased.

When binary mixtures of shallow systems of granular particles are agitated in vertical direction, where the types of particles differ in density or size, segregation is observed [5,6,35,36]. The segregation was attributed to a lack of equipartition of energy between the species, resulting in a larger pressure of the light (or small) particles than of the heavy (large) ones leading to cluster formation of the latter species [36]. As a further effect, in the cluster state it was observed that the heavy particles are characterized by large fluctuations of the horizontal kinetic energy [37]. The phenomena of structure formation in shallow sinusoidally vertically vibrated containers appear to be rather independent of the specifics of driving. Very similar phenomena were found for mechanical driving, e.g. [28,29,34] and for driving through an electrostatic field [31,38]. Only for the case of multiplicative driving of the velocity, a qualitatively different behavior was reported [35], found in MD simulations.

Segregation of particles is also observed when a binary mixture of particles of different sizes, density or surface properties in a shallow container is subjected to *horizontal* vibration [39,40]. Here the phases organize themselves in stripe oriented perpendicular to the direction of shaking. Several different phases can be found in such systems [41,42], dependent on the parameters of driving, the filling height and the particle characteristics. The transitions between the phases can be characterized by a continuous phase transition, including critical slowing down [43]. The pattern coarsens with time and shows similarity to simple models of the geological process of stone striping [26]. The process of stripe formation in horizontally vibrated shallow bidisperse systems can be phenomenologically described by a monodisperse system of particles interacting via an effective attractive short-range anisotropic interaction [44,45].

In view of the large body of literature regarding coarsening and segregation in vertically vibrated (sub-) monolayers and shallow systems discussed briefly above, surprisingly little is known about coarsening and phase segregation in such systems when subjected to horizontal vibrations. According to our knowledge, horizontally vibrated sub-monolayers were only addressed by Strassburger et al. [46] and Betat et al. [47]. Related phenomena of phase separation and segregation are observed for swirling excitation (e.g. circular vibration) of a horizontal sub-monolayer of granular material [48–51] as well as in horizontal containers when one of the side walls vibrates [52].

By now, the reasons for segregation and phase separation phenomena in granular systems are not fully understood and there is no general theory which would allow to reliably predict whether a given granular system under certain external conditions will mix or de-mix. An exception is the force-free granular gas, that is, a dilute system in the absence of external driving where we have some understanding on the mechanism of structure formation, e.g. [53–56].

Besides being of interest by their own, fluidized granular media in shallow agitated containers have been studied extensively since these quasi-two dimensional systems can be easily analysed regarding the collective behavior and the motion of individual grains. Understanding the physics of these systems on the microscopic scale and on the collective scale could lead to a more general theoretical understanding of the collective dynamics of granular media.

The aim of the present paper is to explain pattern formation in horizontally vibrated sub-monolayers of identical, almost spherical particles as a consequence of frustration effects due to friction of the particles.

2 Experiment

2.1 Setup

We investigated structure formation of a sub-monolayer of about 4,800 approximately spherical mustard seeds of radius $r = (0.11 \pm 0.01)$ cm. The particles are confined in an unpolished aluminum box of length 50 cm and width 10 cm. The box was mounted longitudinal on a linear bearing and driven by a motor to subject it to horizontal vibrations. The amplitude and frequency of these sinusoidal oscillations could be adjusted via software. A digital camera was used to observe the system from above using phase-locked imaging via a trigger signal from the motor. As in similar experiments performed by Strassburger et al. [46] and by Betat et al. [47] we observed the rapid formation of a stripe pattern oriented perpendicular to the direction of driving.

Before starting the measurement, the material was liquidized by intense driving (frequency $f = 5$ Hz, amplitude $\mathcal{A} = 2$ cm) to provide a homogeneous particle distribution as initial condition. At time $t = 0$ we lowered the frequency to $f = 3$ Hz ($\omega = 18.9\,\mathrm{s}^{-1}$) and started the recording.

2.2 Results

Within the first few seconds we observed structure formation in the system, see Fig. 1. The initially homogeneous system exhibits pronounced stripes of granular material perpendicular to the direction of the shaking. The first sign of the structures appears in the central area of the container after about 1 second. Subsequently, more clusters appear and spread over the entire area with approximately uniform spacing between the stripes. After about 10 seconds, the system reaches a quasi stationary state with only slow changes over time. This is in good agreement with the findings of Refs. [46,47].

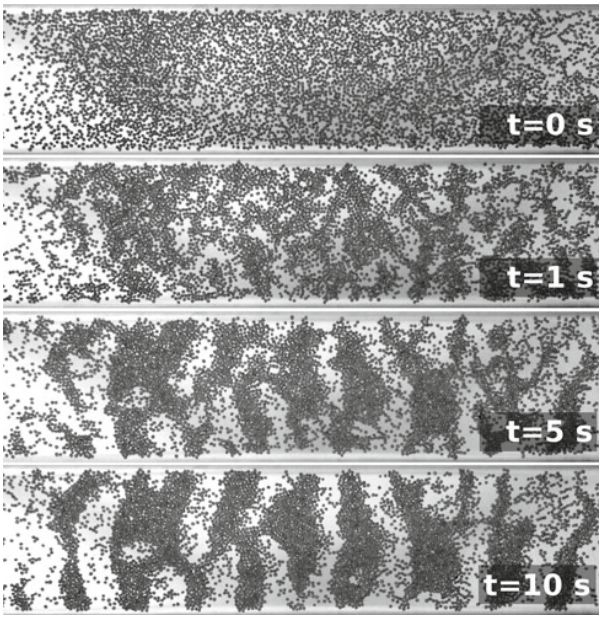

Fig. 1 Sequence of snapshots of the sub-monolayer at $\omega = 18.9\,\mathrm{s}^{-1}$ and $\mathcal{A} = 2$ cm. The pictures are taken at equal phases of the oscillation of the container. First signs of patterns appear at $t \approx 1$ s. At $t = 10$ s the stripe-like structure has fully developed and only minor changes are visible later on. The corresponding simulation is highlighted in Fig. 4

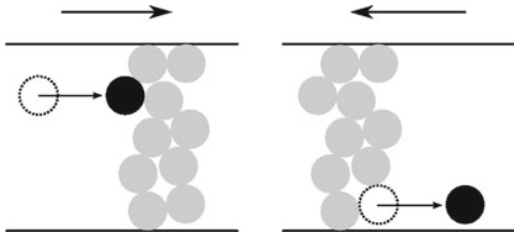

Fig. 2 Isolated particles moving gas-like between the stripes may be captured by the stripe (*left*). When reverting the direction of the oscillatory motion, particles belonging to the bulk of the stripes may sublimate and enter the gas phase (*right*)

When the structure has fully developed, the positions of the stripes relative to the bottom of the container remain mostly invariant. Isolated particles can move between the stripes during the period of the oscillation and behave similar to particles in a granular gas, sloshing back and forth in phase with the oscillation of the container. When the direction of the horizontal oscillation changes, that is, twice per period, particles close to the sides of the stripes facing the incoming wall can leave the stripes to join the free particles in-between, while isolated particles colliding with the stripes can be captured and join the bulk of the stripes (Fig. 2). This may cause variations of the positions and sizes of the stripes.

Following the arguments of Refs. [52,57–60] this may be considered as the result of a frustration effect [61]: Free particles may roll on the horizontal surface of the bottom of the container feeling only the relatively small force of rolling friction. Particles inside the stripes are in contact with the bottom of the container, and additionally with other particles, such that their rolling motion is suppressed due to hindrance by the much larger static friction.

At no point any "jumping" motion of particles was observed, contradicting the main assumption of the modeling of the effect in Ref. [47].

3 Simulation

In Ref. [47] the stripe formation was explained by means of a Cellular Automaton model. The main assumption of this model was that during the periodic motion of the container, the particles are lifted and perform a ballistic flight, thus, jump from their current stripe to an adjacent one.

As we do not observe any jumping particles in the experiment, this kind of motion may be excluded as the origin of pattern formation. In order to obtain an alternative explanation, we performed Molecular Dynamics simulations of the system. We take into account both normal and tangential forces to model the particle-particle and particle-wall interaction.

3.1 Forces

For two spherical particles i and j centered at $\mathbf{r}_i$ and $\mathbf{r}_j$, the normal vector is defined as the unit vector $\hat{\mathbf{r}} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j|$. The interaction forces for particles in contact are split into a normal component along $\hat{\mathbf{r}}$ and the tangential component. For the normal part, a viscoelastic Hertz interaction force F^n is assumed [62],

$$F^n = \frac{2\,Y\sqrt{R_{\mathrm{eff}}}}{3\left(1-\nu^2\right)}\left(\xi^{3/2} + \frac{3}{2}A\sqrt{\xi}\frac{\mathrm{d}\xi}{\mathrm{d}t}\right), \tag{1}$$

where $R_{\mathrm{eff}} = \frac{R_i R_j}{R_i + R_j}$ is the effective radius of the colliding particles and $\xi(t) = R_i + R_j - \left|\mathbf{r}_i(t) - \mathbf{r}_j(t)\right|$ characterizes the deformation of the particles. Y, ν and A are the Young's modulus, the Poisson's ratio and the viscoelastic dissipative constant, respectively.

For the tangential component we use the model by Haff and Werner [63]

$$F^t = -\mathrm{sgn}(v^t_{\mathrm{rel}})\,\min\left(\gamma^t\left|v^t_{\mathrm{rel}}\right|, \mu\left|F^n\right|\right), \tag{2}$$

where γ^t is the tangential damping constant, μ the coefficient of the Coulomb friction and v^t_{rel} the relative surface velocity at the point of contact. In addition to a force acting on the center of mass of each particle, the tangential force also generates a torque. The forces between a particle i and a wall of the container are calculated according to the same force laws

Table 1 Material parameters used in the Molecular Dynamics simulation

Material density ρ	$2.5 \times 10^3\,\mathrm{kg/m^3}$
Young's modulus Y	$10^6\,\mathrm{Pa}$
Poisson's ratio ν	0.3
Dissipative constant A	$8.5 \times 10^{-6}\,\mathrm{s}$
Coulomb friction coefficient μ	0.7
Tangential dissipative constant γ^t	$7 \times 10^{-3}\,\mathrm{Ns/m}$

by having R_j tend towards infinity. As the motion of the particles relative to the floor can be a combination of sliding and rolling, in addition to the sliding friction modelled by F^t, a torque based on the rolling motion of the particles is introduced:

$$\mathbf{M} = -A \,|\, (\hat{\mathbf{r}} \cdot R_i) \times \boldsymbol{\omega}|\, F^n \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|}, \tag{3}$$

where A denotes the viscoelastic dissipative constant and $\boldsymbol{\omega}$ the angular velocity of the particle [64,65].

The numerical parameters used in the simulation, corresponding to the rather soft mustard seeds, are summarized in Table 1.

3.2 Simulation setup

The rectangular container of length 40 cm, width 2 cm and arbitrary height is shaken longitudinal along which periodic boundary conditions are used. The bottom and side walls of the container are solid walls. The radii of the 4,000 particles in the system are sampled from a uniform distribution in the interval [0.025, 0.05] cm and their initial positions are homogeneously distributed on the bottom of the container. Initially, the particles are assigned uniformly distributed random velocity components in the plane from the interval $[-0.5, 0.5]$ cm/s. The initial vertical velocity was zero. For homogenization we let the system equilibrate for 10 s. Afterwards all velocities (linear and angular) and accelerations are reset to zero. The resulting positions are used as initial conditions for the actual simulation. If not mentioned otherwise, the frequency of the shaking is set to an angular frequency $\omega = 20\,\mathrm{s}^{-1}$ (i.e. $f \approx 3.2$ Hz) and the amplitude to $\mathcal{A} = 2.5$ cm for all simulation runs.

3.3 Long-term evolution

We simulate the system for a total of 2,500 s to validate the stability of the simulation and to ensure the system reached a steady state after a few seconds with no further transient behaviour. From the particle trajectories obtained as a result of the MD simulation, we compute the density profile of the particles in the direction of shaking, see Fig. 3.

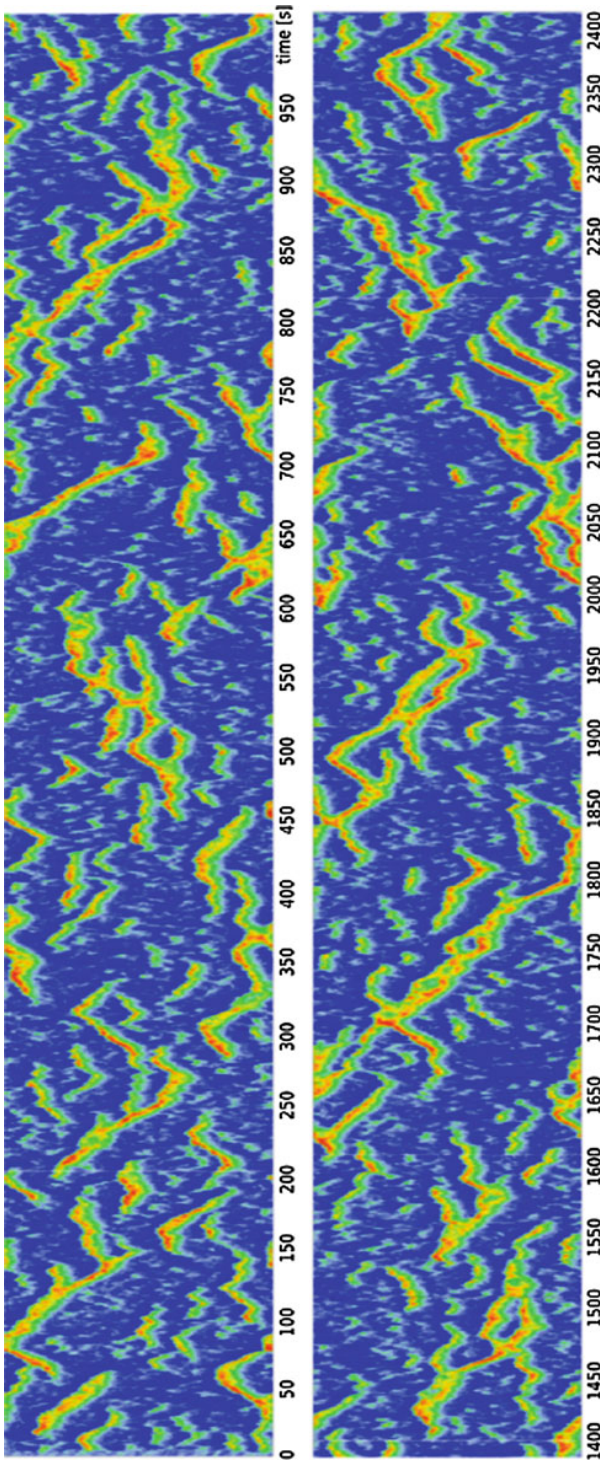

Fig. 3 Evolution of the stripe-like pattern of the density field. The *vertical axis* shows time. The *left side* of the figure shows the evolution for the interval [0, 1,000] s, the right side shows the interval [1,400, 2,400] s. The color code ranges from *blue* (no particles) to *red* (close packing), for the scale see Fig. 5 (color figure online)

As in the experiment, stripes form rather rapidly after a few seconds. We also observe spontaneous appearing and disappearing of high density regions as well as merging and splitting of such regions. As a characteristic feature, we observe that there is always at least one major stripe in the system, accompanied by several minor clusters.

Apart from providing confidence that our simulation captures the essential features of the experiment described above, here we shall not discuss the dynamics of the stripe pattern in more detail. Being beyond the scope of the present paper these results will be published elsewhere.

4 Instability of the homogeneous state

In order to determine the conditions under which stripe formation occurs, we varied the frequency, f, and amplitude, $\mathcal{A}$. Fig. 4 (top) shows density plots obtained from MD simulations for a variety of combinations $(f, \mathcal{A})$. Each of the 54 sub-figures shows the density profile in the direction of shaking in the interval $t = [0, 15]$ s, where the density was computed from the particle positions. The lower part of Fig. 4 provides an abstract overview over regions of different characteristics in the parameter space to be detailed below.

In the parameter space $(f, \mathcal{A})$ we identify three major distinct regions and a transition region:

(a) static region: for weak forcing (low frequency and/or low amplitude) the particles essentially rest on the base plate. No stripe formation is observed and the density plot is almost invariant in time. Small fluctuations of density due to the initial conditions are preserved in this region, resulting in a vertical texture of the density vs. time plots. This region is indicated by blue shading in Fig. 4 (bottom).

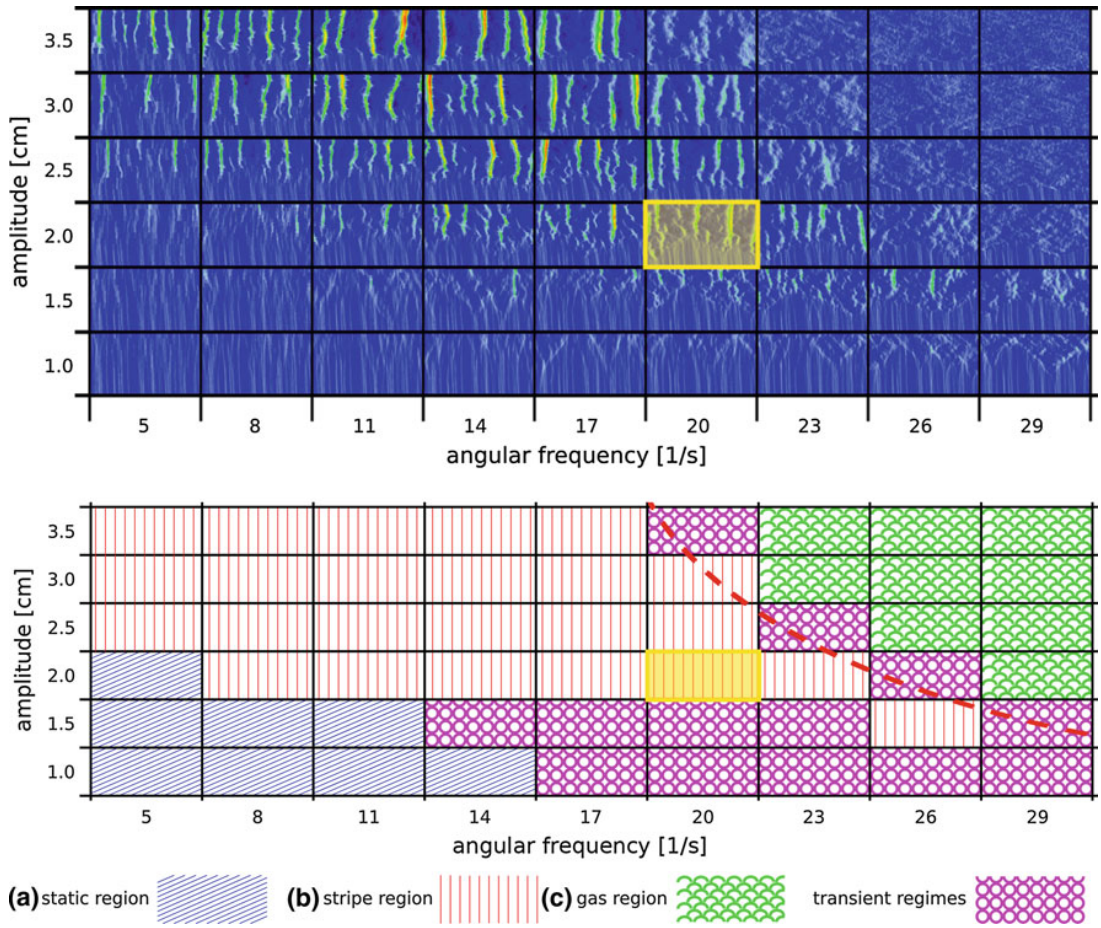

Fig. 4 Patterns resulting from certain combinations of amplitude and frequency of the oscillation. *Top* density profiles versus time for $t = [0, 15]$ s (same type of plot as shown in Fig. 5) for different sets of parameters. *Bottom* abstract representation of the data shown in the *top figure* to emphasize the different regions in the parameter space. The *dashed line* separating the stripe region from the gas region shows the function $\mathcal{A}\,\omega^2/g = c\,\mu$ with $c \approx 1.3$ as determined by fitting (see Sect. 5.3 for discussion). The experiment corresponding to the *highlighted parameter* set is shown in Fig. 1 (color figure online)

(b) stripe region: in a certain range of the (f, $\mathcal{A}$) plane corresponding to intermediate forcing, starting from a homogeneous distribution of particles, stripes emerge after typically 5 s. In Fig. 4 (bottom) this range is indicated by red shading.
(c) gas region: for strong forcing, that is, large amplitude and/or frequency, the system behaves gas-like. Here the particles move irregularly resulting in a homogeneous density. In difference to region (a), here the small time-invariant fluctuations of the density due to the initial conditions (visible as weak vertical texture) are not preserved. The gas region is indicated by green shading in Fig. 4 (bottom).
(d) transition region: the transition between the stripe region (b) and the static region (a) (towards weak forcing) and the gas region (c) (towards strong forcing) is not sharp. In the transition region, pink shading in Fig. 4 (bottom), short-lived stripe patterns appear and disappear dynamically. This region can, thus, not be clearly attributed to one of the major regions (a), (b) and (c).

Stripe formation can be noticed at intermediate driving frequencies and intermediate to large amplitudes. In this region in the (f, $\mathcal{A}$) parameter space, the stripe pattern is essentially stable during the entire simulation. While fluctuations of the density pattern are noticeable they are small such that the number and position of the stripes are essentially invariant.

5 Mechanism of the stripe formation

In order to identify the mechanism which is responsible for the formation of the stripe pattern, we performed MD simulations starting with the setup described in Sect. 3.2 as a reference system (referred to as *full-fledged system*). In the subsequent steps we modified this setup with respect to the boundary conditions and the interaction forces. The results of the simulations were then compared to the results due to the reference system.

Five different setups were investigated:

(a) Full-fledged simulation (see Sect. 3.2 for the description)
(b) System with periodic boundary conditions in the direction perpendicular to the vibration
(c) System with suppressed friction of the particle-particle contact
(d) System with suppressed vertical coordinate (effective 2D-system)
(e) System of elastic particles

All simulated systems (b)–(e) are derived from the full-fledged simulation (a) but with particular modifications. Thus, all properties and system parameters of the cases (b)–(e) are the same as for case (a), except for those wich are explicitly mentioned. For each of the cases, we performed simulations starting with different initial distributions of the particles and obtained results very similar to the examples presented here.

5.1 Setup (a): Full-fledged simulation

As a reference system to compare with, we first run the simulation using the full force models as described in Sect. 3.2 for 20 s.

Figure 5 shows the evolution of the density profile along the direction of the oscillation. Time progresses in vertical direction, $t = (0 \ldots 20)$ s. The spontaneous formation of stripes starts about (5 ± 1) s after the simulation was started, depending on the concrete initial positions of the particles. The initial distribution of the particles influences the number, positions, sizes and orientations of the stripes.

5.2 Setup (b): Periodic boundary conditions

To test for the influence of the side walls, we replace the walls of the container in the direction perpendicular to the oscillatory motion by periodic boundary conditions. We find that the stripe formation occurs after about (5 ± 1) s (Fig. 6b), that is, slightly later than in the reference case. We observe a larger number of slightly smaller stripes than in the reference system. Compared with the reference system (a), we notice also a larger number of transient stripes which appear and disappear in the course of time. It can be concluded, that walls are not critical for stripe formation and the difference in the width of the container between the simulation and the experiment is of minor importance. The additional damping caused by the particle-wall friction may abet stripe formation, but it is not necessary for it. The overall characteristics of the density plot are rather close to the reference system, case (a), see Fig. 5.

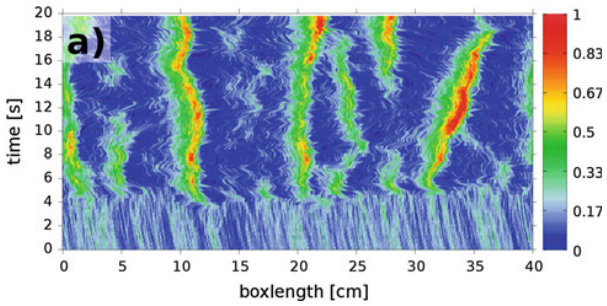

Fig. 5 Particle number density in the direction of shaking as obtained from MD simulation at $\omega = 20\,\mathrm{s}^{-1}$ and $\mathcal{A} = 2.5$ cm. The *vertical axis* shows time. *Blue* color encodes density zero (no particles), *red* encodes the density corresponding to close packing (color figure online)

5.3 Setup (c): frictionless particles

Choosing $\mu = 0$ for the Coulomb friction parameter effectively turns off friction, see Eq. (2). In setup (c) we simulate particles which interact frictionless with other particles ($\mu = 0$) but *not* with the container walls.

The resulting density profile obtained from the MD simulation of the system (Fig. 6c) does not reveal any sign of stripe formation. Instead, during the entire simulation time the density profile stays approximately homogeneous with some fluctuations that are always present in a disordered many-body system. The slight inclination of the initial random pattern to the left is caused by the direction of the initial thrust of the shaking. It fades away during the first seconds of the simulation. Note that this inclination is noticeable in all density plots in Figs. 5 and 6, albeit not as clearly as in the present case.

To elucidate the idea of the somewhat unphysical assumption of frictionless particle-particle contact and at the same time frictional particle-wall contact, let us consider the transfer of energy between the externally driven container and the particles inside the container. In contrast to systems with vertical driving, here the only mechanism of energy transfer is via tangential (frictional) forces (remember that we have periodic boundary conditions in the direction of shaking). If we would switch off friction between the floor and the particles, the particles would not receive any energy from the driving and stay in their initial position.

Given energy transfer via friction, an isolated particle feels a tangential force from the floor resulting in two different modes of motion: (a) the particle feels a torque and changes its angular velocity and (b) the particle is accelerated because of the linear force. The particle resists against rolling through its moment of inertia. A particle having a larger moment of inertia while keeping the mass constant, would hardly change its angular velocity but significantly change its linear velocity. On the other hand a particle with a smaller moment of inertia would instead change its angular velocity but not significantly its linear velocity. Consequently, any mechanism counteracting rolling would lead to enhanced transfer of energy to linear motion.

Such a counteracting mechanism may be established when a particle is not isolated but in frictional contact with other particles. In this case, the frictional forces with other particles may lead to a torque opposite to the torque caused by the moving floor.

Thus, frictional contact with another particle has a similar effect as an increased moment of inertia, namely an enhanced transfer of energy to the linear motion. Indeed, since the external driving acts on both contacting particles in the same way, we find a situation of frustration sketched in Fig. 7.

If the particles would both roll on the floor (no sliding), inevitably the particles must slide at their point of contact as here the vectors of velocity of the surfaces point in opposite directions. This in turn would lead to an apparently increased

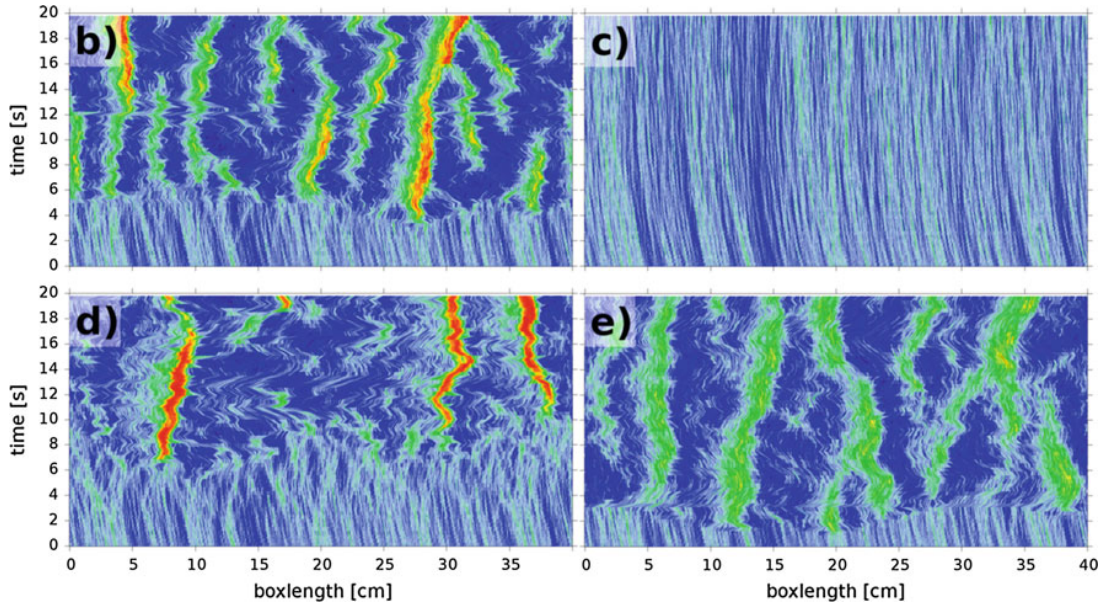

Fig. 6 Evolution of the particle number density obtained from MD simulations. The 4 subfigures correspond to the systems (**b**)–(**e**) described in the beginning of Sect. 5, namely: (**b**) system with periodic boundary conditions in both horizontal directions, (**c**) frictionless particles, (**d**) suppressed motion in vertical direction, and (**e**) elastic particles. Apart from the named modifications, all other system specifications are identical to the reference system described in Sect. 5.1. Color coding is the same as in Fig. 5 (color figure online)

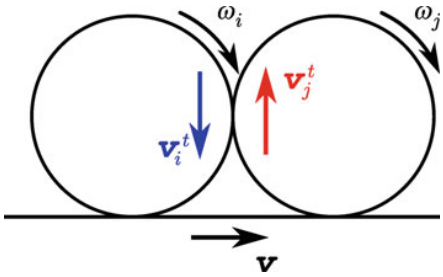

Fig. 7 Sketch of the frustrated situation of two particles in frictional contact rolling on the floor. See the text for discussion

moment of inertia regarding the contact with the floor and to an enhanced transfer of energy of the linear motion.

The described mechanism established an important difference of the energy transfer and, thus, the motion between particles located in the stripes and particles moving in between the stripes: particles in the stripes are in frictional contact with their neighbors and according to the arguments above less mobile with respect to the moving floor and consequently also with respect to one another.

This situation is somewhat similar to the formation of clusters in a dilute force-free granular gas. Here clustering is attributed to a pressure instability due to increased dissipation rate in regions of increased density [53], that is, smaller relative mobility. Related separation of granular gases in dense and dilute regions due to pressure instabilities are also observed in driven gas systems, e.g. [66–68] including vertically vibrated shallow layers which also reveals instabilities due to negative compressibility of the effective fluid [69].

The above arguments would support the hypothesis that tangential forces, that is, friction is responsible for the stripe formation since it leads to the mentioned frustration with the consequence that particles in dense regions are less mobile with respect to their neighbors. This hypothesis can be tested by disregarding the friction between the particles but not between the particles and the floor. By this, we suppress the frustrated situation and the pressure instability should be avoided.

Our results presented in Fig. 6c support this hypothesis: With disabled sliding friction between particles, no jamming of the particles is observed and the system behaves largely like independent spheres.

Exploiting the frustration argument we can motivate the border between the stripe region and the gas region in the $(\mathcal{A}, \omega)$-plane, see Fig. 4. Obviously, the frustration effect ceases when particles in contact can slide collectively on the floor. This is the case when the maximal tangential force due to the driving, $m\,\mathcal{A}\,\omega^2$, exceeds the maximal tangential force F^t the particles can afford to resist sliding which in turn is related to the normal force $F^n = m\,g$ via Coulomb's friction law, $F^t = \mu\,F^n$. Consequently, the frustration effect which is responsible for the stripe formation ceases when

$$\mathcal{A}\,\omega^2 = \frac{F^t}{m} = \mu\,\frac{F^n}{m} = \mu\,g\,, \tag{4}$$

$$\frac{\mathcal{A}\,\omega^2}{g} \equiv \Gamma = \mu. \tag{5}$$

Note that the above condition assures that the particles can slide in singular points in time during the oscillation $x = \mathcal{A}\,\cos(\omega\,t)$, when $\omega\,t = n\,\pi$. To have a noticeable effect on the dynamics, the sliding condition needs to be fulfilled at least for a certain non-negligible part of the period, that is, instead of the above condition we better write $\mathcal{A}\,\omega^2/g = c\,\mu$. By fitting the latter function to match the transition region in Fig. 4 (dashed line), we estimate $c \approx 1.3$.

Note that the situation in our system is similar to a horizontally shaken sub-monolayer of particles differing in their coefficient of friction having a similar effect regarding the mobility. Here one observes "segregation by friction" [57,58], which had been attributed to frustration effects, too. In such a frustrated situation one or more of the contacts of the particle (with the floor and its neighbors) must turn from rolling to sliding to overcome a kind of jamming situation.

It was shown that such frustration effects may lead to interesting phenomena, even in one-dimensional systems when cylinders roll on a plane while being in contact [61]. In systematic experiments [52,59] and simulations [60,70] of periodically driven and un-driven systems it was found that the effect of rolling is much smaller than the effect of sliding, for the system dynamics.

5.4 Setup (d): effective 2D-Simulation

In both our experimental work and our simulations we did not notice significant dynamics with regard to the vertical direction. This contradicts the work by Strassburger et al. [46] and Betat et al. [47] who explicitly assumed that the particles are jumping. This assumption is the main foundation of their mathematical description using a Cellular Automaton model.

Of course, in any three-dimensional system there is always *some* dynamics in any direction of space. Therefore, to definitely exclude that jumping of particles might be responsible for stripe formation, we modified the used force model by setting all forces for inter-particle collisions to zero in the vertical direction.

The corresponding density profile, Fig. 6d, does reveal stripes in agreement with our experimental observation, Fig. 1, and also the reference setup, see Fig. 5.

Note that restricting the model to a 2D system in this way, introduces a strong idealisation of the model since, obviously, it does not comply with Newton's equation of motion. Consequently, we must not take the results too literally.

In particular we should refrain from discussing any quantitative features of the density plot.

5.5 Setup (e): elastic particles

Finally let us consider the influence of dissipation to stripe formation. To this end, we replace the dissipative normal force, Eq. (1) by the pure Hertz force, that is, $A = 0$. Note that there is still dissipation according to the damping of the tangential force via γ^t, consequently, we do not observe unlimited increase of the kinetic energy. The corresponding density plot, Fig. 6e, reveals stripes which differ in shape and size from the reference setup but appear qualitatively similar otherwise.

6 Conclusion

We investigated a horizontally vibrated granular submonolayer of almost spherical particles confined in a rectangular container and found self-organized stripe patterns in agreement with earlier results [46,47]. We reproduced the experimental observation in MD simulations to obtain qualitative agreement with the experiment. As in the experiment, the pattern varies dynamically in the course of time, that is, number and position of the stripes are subject to random fluctuations. Stripes, that is, regions of large particle number density may spontaneously appear and disappear as well as merge and disintegrate. Nevertheless, some features of the pattern stay invariant, for example the typical spacing of the stripes and their approximate number.

In [46,47] the system was modeled by means of a Cellular Automaton (CA) with the main assumption that during the oscillation period the particles jump from one stripe (region of enhanced density) to the next. However, neither in the experiment nor in the simulation we could find any evidence of particles being significantly lifted from the floor. Even if we completely suppress motion in vertical direction in the simulation, formation of stripes occurs. Consequently, we believe that the mentioned CA model provides an inadequate description of the phenomenon.

In order to identify the origin of pattern formation, we performed MD simulations of modified systems where we systematically excluded possible scenarios. This way, we excluded that (reflective) boundary conditions, dissipative properties and the dynamics of particles in vertical direction are essential for the formation of stripes. Comparing these modified systems with the full-fledged reference system, in all cases we could clearly observe stripe formation with only quantitative (small) modifications of their characteristics.

Only suppression of the frictional particle-particle interaction led to simulation results where the initial homogeneous distribution of particles remained stable, that is, no pattern formation could be observed in this case. Based on this observation, we proposed an explanation of the mechanism of pattern formation based on frustration of particles in dense regions who are simultaneously in contact with the floor and one or more of their neighbors. Such a frustration scenario and pattern formation originating from it was reported also for one-dimensional few-particle systems [61]. The frustrated dynamics leads effectively to an enhanced friction coefficient of particles in dense regions (stripes) as compared to particles in dilute gaseous regions.

While friction is essential to the effect reported here, we wish to mention that also pure dissipation without friction would lead to an inhomogeneous density distribution. In this case, the collisions of particles with the moving walls would lead to an raised granular temperature in the vicinity of the walls. Since in the absence of friction, the side walls are the only sources of energy, temperature and, thus, pressure decrease with distance from the walls. Consequently there is a permanent pressure pushing the material away from the walls and as a result, asymptotically, density would monotonously increase with distance from the wall. The asymptotic stationary state would have a single stripe in the center of the box. In the case of frictional particles, the described frustration scenario causes input of energy also from the bottom wall via tangential forces leading, consequently, to the characteristic stripe pattern.

We believe that the presented results might lead also to a better understanding of the dynamics of horizontally agitated sub-monolayers under swirling excitation [48–51]. In these systems, dynamical transitions between gas-like and solid-like behavior as well as pronounced segregation phenomena are observed. In view of the results presented here, these effects may also be caused mainly by hindrance of rolling motion when particles are in frictional contact. Despite of the plain existence of a region of increased density in the center of the container which may be explained based on kinetic theory [66,67], the mechanism of segregation and phase separation in these systems is still not sufficiently understood.

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References

1. Cooke, M.H., Stephens, D.J., Bridgwater, J.: Powder mixing—a literature survey. Powder Technol. **15**, 1–20 (1976)
2. Kudrolli, A.: Size separation in vibrated granular matter. Rep. Prog. Phys. **67**, 209–247 (2004)
3. Pöschel, T.: Dynamik Granularer Systeme. Logos, Berlin (2001)
4. Ristow, G.: Pattern Formation in Granular Materials, volume of 164 of Springer Tracts in Modern Physics, Springer (1999)

5. Aranson, I.S., Tsimring, L.S.: Patterns and collective behaviour in granular media: theoretical concepts. Rev. Mod. Phys. **78**, 641–692 (2006)
6. Aranson, I.S., Tsimring, L.S.: Granular Patterns. Oxford University Press, Oxford (2009)
7. Allen, J.R.: Asymmetrical ripple marks and the origin of cross-stratification. Nature **194**, 167–169 (1962)
8. Anderson, R.S., Bunas, K.L.: Grain size segregation and stratigraphy in aeolian ripples modelled with a cellular automaton. Nature **365**, 740–743 (1993)
9. Koeppe, J., Enz, M., Kakalios, J.: Avalanche segregation of granular media. In: Behringer, R.P., Jenkins, J.T. (eds.) Powders and Grains'97, pp. 443–446. Rotterdam, Balkema (1997)
10. Julien, P.Y., Lan, Y.Q., Raslan, Y.: Experimental mechanics of sand stratification. In: Behringer, R.P., Jenkins, J.T. (eds.) Powders and Grains'97, pp. 487–490. Rotterdam, Balkema (1997)
11. Makse, H.A., Havlin, S., King, P.R., Stanley, H.E.: Novel pattern formation in granular matter. In: Schimansky-Geier, L., Pöschel, T. (eds.) Stochastic Dynamics. Lecture Notes in Physics, pp. 319–333. Springer, Berlin (1997)
12. Makse, H.A.: Stratification instability in granular flows. Phys. Rev. E **56**, 7008–7016 (1997)
13. Makse, H.A., Havlin, S., King, P.R., Stanley, H.E.: Spontaneous stratification in granular mixtures. Nature **386**, 379–382 (1997)
14. Makse, H.A., Havlin, S., Ivanov, PCh., King, P.R., Prakash, S., Stanley, H.E.: Pattern formation in sedimentary rocks: connectivity, permeability, and spatial correlations. Phys. A **233**, 587–605 (1996)
15. Makse, H.A., Cizeau, P., Stanley, H.E.: Possible stratification mechanism in granular mixtures. Phys. Rev. Lett. **78**, 3298–3301 (1997)
16. Makse, H.A., Cizeau, P., Stanley, H.E.: Modeling stratification in two-dimensional sandpiles. Phys. A **249**, 391–396 (1998)
17. Makse, H.A., Herrmann, H.-J.: Microscopic model for granular stratification and segregation. Europhys. Lett. **43**, 1–6 (1998)
18. Gray, J.M.N.T., Hutter, K.: Pattern formation in granular avalanches. Cont. Mech. Thermodyn. **9**, 341–345 (1997)
19. Gray, J.M.N.T., Hutter, K.: Physik granularer Lawinen. Physikalische Blätter **54**, 37–43 (1998)
20. Grasselli, Y., Herrmann, H.-J.: Experimental study of granular stratification. Granul. Matter **1**, 43–47 (1998)
21. Boutreux, T., De Gennes, P.G.: Surface flow of granular mixtures. In: Behringer, R. P., Jenkins, J. T. (eds.) Powders and Grains'97, pp. 439–442. Rotterdam, Balkema (1997)
22. Boutreux, T., De Gennes, P.G.: Surface flows if granular mixtures: I. General principles and minimal model. J. Physique I **6**, 1295–1304 (1996)
23. Meakin, P.: A simple two-dimensional model for particle segregation. Phys. A **163**, 733–746 (1990)
24. Sorby, H.C.: On the structures produced by the currents present during the deposition of stratified rocks. Geologist **2**, 137–147 (1859)
25. Head, D.A., Rodgers, G.J.: Slowly driven formation with granular mixtures. Phys. Rev. E **56**, 1976–1980 (1997)
26. Werner, B.T., Hallet, B.: Numerical simulation of selforganized stone stripes. Nature **361**, 142–145 (1993)
27. Washburn, A.L.: Classification of patterned ground and review of suggested origins. Geol. Soc. Am. Bull. **67**, 823–866 (1956)
28. Olafsen, J.S., Urbach, J.S.: Clustering, order, and collapse in a driven granular monolayer. Phys. Rev. Lett. **81**, 4369–4372 (1998)
29. Prevost, A., Melby, P., Egolf, D.A., Urbach, J.S.: Nonequilibrium two-phase coexistence in a confined granular layer. Phys. Rev. E **70**, 050301(R) (2004)
30. Melby, P., Vega Reyes, F., Prevost, A., Robertson, R., Kumar, P., Egolf, D.A., Urbach, J.S.: The dynamics of thin vibrated granular layers. J. Phys.: Condens. Matter **17**, S2689–S2704 (2005)
31. Sapozhnikov, M.V., Aranson, I.S., Olafsen, J.S.: Coarsening of granular clusters: two types of scaling behaviors. Phys. Rev. E **67**, 010302(R) (2003)
32. Losert, W., Cooper, D.G.W., Gollub, J.P.: Propagating front in an excited granular layer. Phys. Rev. E **59**, 5855–5861 (1999)
33. Olafsen, J.S., Urbach, J.S.: Velocity distributions and density fluctuations in a granular gas. Phys. Rev. E **60**, R2468–R2471 (1999)
34. Nie, X., Ben-Naim, E., Chen, S.Y.: Dynamics of vibrated granular monolayers. Europhys. Lett. **51**, 679–684 (2000)
35. Cafiero, R., Luding, S., Herrmann, H.-J.: Two-dimensional granular gas of inelastic spheres with multiplicative driving. Phys. Rev. Lett. **84**, 6014–6017 (2000)
36. Rivas, N., Cordero, P., Risso, D., Soto, R.: Segregation in quasi-two-dimensional granular systems. New J. Phys. **13**, 055018 (2011)
37. Rivas, N., Ponce, S., Gallet, B., Risso, D., Soto, R., Cordero, P., Mujica, N.: Sudden chain energy transfer events in vibrated granular media. Phys. Rev. Lett. **106**, 088001 (2011)
38. Aranson, I.S., Blair, D., Kalatsky, V.A., Crabtree, G.W., Kwok, W.-K., Vinokur, V.M., Welp, U.: Electrostatically driven granular media: phase transitions and coarsening. Phys. Rev. Lett. **84**, 3306–3309 (2000)
39. Mullin, T.: Coarsening of self-organized clusters in binary mixtures of particles. Phys. Rev. Lett. **84**, 4741–4744 (2000)
40. Mullin, T.: Mixing and de-mixing. Science **295**, 1851 (2002)
41. Reis, P.M., Ehrhardt, G., Stephenson, A., Mullin, T.: Gases, liquids and crystals in granular segregation. Europhys. Lett. **66**, 357–363 (2004)
42. Reis, P.M., Sykes, T., Mullin, T.: Phases of granular segregation in a binary mixture. Phys. Rev. E **74**, 051306 (2006)
43. Reis, P., Mullin, T.: Granular segregation as a critical phenomenon. Phys. Rev. Lett. **89**, 244301 (2002)
44. Pica Ciamarra, M., Coniglio, A., Nicodemi, M.: Dynamically induced effective interaction in periodically driven granular mixtures. Phys. Rev. Lett. **97**, 038001 (Jul 2006)
45. Pica Ciamarra, M., Coniglio, A., Nicodemi, M.: Phenomenology and theory of horizontally oscillated granular mixtures. Eur. Phys. J. E. **22**, 227–234 (2007)
46. Strassburger, G., Betat, A., Scherer, M.A., Rehberg, I.: Pattern formation by horizontal vibration of granular material. In: Wolf, D.E., Schreckenberg, M., Bachem, A. (eds.) Workshop on Traffic and Granular Flow, pp. 329–334. World Scientific, October 9–11 (1995)
47. Betat, A., Dury, C., Rehberg, I., Ristow, G., Scherer, M., Schröter, M., Straßburger, G.: Formation of patterns in granular materials. In: Busse, F.H., Müller, S.C. (eds.) Evolution of Spontaneous Structures in Dissipative Continuous Systems, pp. 495–545. Springer, Berlin (1998)
48. Scherer, M.A., Buchholtz, V., Pöschel, T., Rehberg, I.: Swirling granular matter: from rotation to reptation. Phys. Rev. E **54**, R4560–R4563 (Nov 1996)
49. Scherer, M.A., Kötter, K., Markus, M., Goles, E., Rehberg, I.: Swirling granular solidlike clusters. Phys. Rev. E **61**, 4069–4077 (Apr 2000)
50. Aumaître, S., Kruelle, C.A., Rehberg, I.: Segregation in granular matter under horizontal swirling excitation. Phys. Rev. E **64**, 041305 (2001)
51. Schnautz, T., Brito, R., Kruelle, C.A., Rehberg, I.: A horizontal brazil-nut effect and its reverse. Phys. Rev. Lett. **95**, 028001 (2005)
52. Kudrolli, A., Wolpert, M., Gollub, J.P.: Cluster formation due to collisions in granular material. Phys. Rev. Lett. **78**, 1383–1386 (1997)
53. Goldhirsch, I., Zanetti, G.: Clustering instability in dissipative gases. Phys. Rev. Lett. **70**, 1619–1622 (1993)
54. Brilliantov, N.V., Salueña, C., Schwager, T., Pöschel, T.: Transient structures in granular gases. Phys. Rev. Lett. **93**, 134301 (2004)